S.Y. B. SC. (COMPUTER SCIENCE) SEM –III (CBCS - 2016 COURSE) : WINTER - 2017

SUBJECT: LINEAR ALGEBRA

Day: Tuesday
Date: 31/10/2017

Time: 11.00 A.M. TO 02.00 PM
Max. Marks: 60

W-2017-0721

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Let V be a vector space and W be its non-empty subset. Then W is a subspace of v if and only if it is closed under addition and scalar multiplications on i.e.:
i) if $u, v \in W$ then $u+v \in W$
ii) if $u \in W$ and $k \in \mathbb{R}$ *then* $ku \in W$

b) Show that the set $S = \{(1,2,1),(2,9,0),(3,3,4)\}$ is linearly independent in $\mathbb{R}^3$.

c) Determine whether or not S is basis for $V = \mathbb{R}^3$ $S = \{(1,1,1),(2,2,0),(3,0,0)\}$

Q.2 Attempt any **TWO** of the following: **(12)**

a) Prove that if λ is an eigen values of a square matrix A, then λ^m is an eigen value of A^m for every positive integer m.

b) Find all the eigen values of A and find the eigenspace. Corresponding to the smallest and largest eigen value of matrix A where, $A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}$

c) Find matrix P that diabgonalizes the matrix, $A = \begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 4 & 5 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

Q.3 Attempt any **TWO** of the following: **(12)**

a) Let T: $V \to$ W be a linear transformation then prove that,
i) The kernel of T is a subspace of V.
ii) The range of T is a subspace of W.

b) Find standard matrix for linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^5$ defined as,
$T(x,y,z) = (2x+y-z, -y+2z, x-z, x+y-z, 2x)$.

c) Let $T:\mathbb{R}^3 \to \mathbb{R}^2$ be linear transformation defend by
$T(x,y,z) = (x+y+z, x-y-z)$. Find the basis for Kernal of T and basis for range of T. also verify rank-nullity theorem.

P. T. O.

Q.4 Attempt any **THREE** of the following: **(12)**

a) For what value of 'a' does the following system has:

i) Unique solution or trivil solution

ii) Infinitely many solutions.

$$(a-3)x + y = 0$$
$$x + (a-3)y = 0$$

b) Find and LU factorization of the coefficient matrix of the given linear system $A\overline{X} = \overline{b}$ solve the linear system using a forward substitution followed by a back substitution.

$$A = \begin{bmatrix} 4 & 2 & 3 \\ 2 & 0 & 5 \\ 1 & 2 & 1 \end{bmatrix}, \quad \overline{b} = \begin{bmatrix} 1 \\ -1 \\ -3 \end{bmatrix}$$

c) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2×2 matrix, then show that if A is invertible, then $ad - bc \neq 0$ and in this case $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

d) Solve the following system by Gauss Jordan elimination method:

$$x_1 + 3x_2 + x_3 + x_4 = 3$$
$$2x_1 - 2x_2 + x_3 + 2x_4 = 8$$
$$3x_1 + x_2 + 2x_3 - x_4 = -1$$

Q.5 Attempt any **FOUR** of the following: **(12)**

a) State **True** or **False**

"State system of n linear equations in n unknowns has unique solution". Justify your answer.

b) Find all values of x such that $\overline{v}.\overline{v} = 1$ where $\overline{v} = \begin{bmatrix} 1/2 \\ -1/2 \\ x \end{bmatrix}$

c) Show that characteristics equation of 2×2 matrix A is given by, $\lambda^2 - tr(A)\lambda + \det(A) = 0$ where *tr(A)* is a trace of A which is defined as sum of elements on the main diagonal.

d) Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$. Find the eigen values of A^4.

e) Define: i) Vector space

ii) Subspace of a vector space

f) Define: i) Kernel of linear transformation

ii) range of linear transformation

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