F.Y. B. SC. (COMPUTER SCIENCE) SEM – I (CBCS - 2016 COURSE) : WINTER - 2017

SUBJECT: ALGEBRA

Day: Wednesday
Date: 01/11/2017

W-2017-0703

Time: 11.00 A.M. TO 02.00 PM
Max. Marks. 60

N.B.:

1) All Questions are **COMPULSORY.**
2) Figures to the **RIGHT** indicates full marks.

Q.1 A) Select the correct alternative **(06)**

a) The bijective function means a function is ___________

i) only onto ii) only one-one
iii) Neither onto nor one-one iv) Both one-one and onto

b) Euclid's lemma states if p is a prime integer a, b $\in$ Z then $p|ab \Rightarrow$

i) $p \nmid a$ or $p \nmid b$ ii) $p|a$ and or $p|b$
iii) $p \nmid a$ and $p \nmid b$ iv) none of these

c) In Z_3, $(\overline{10}+\overline{1})=$ _____

i) $\overline{0}$ ii) $\overline{1}$
iii) $\overline{2}$ iv) None of these

d) Real part of Z = 7i is _______

i) 0 ii) 1
iii) 7 iv) -1

e) Modulus of complex number is defined as

i) $r = \sqrt{x^2 + y^2}$ ii) $r = x^2 + y^2$
iii) $r = \sqrt{x^2 - y^2}$ iv) None of these

f) Least common multiple of (60, 80) is _____

i) 120 ii) 60
iii) 180 iv) 240

g) M (R) = 1 if and only if R = ______

i) $\frac{A}{B}$ ii) AB
iii) A-B iv) A $\times$ B

B) Attempt all the following:

a) Define partial order relation.

b) Check whether the function $f(x) = x^2$ is onto or not.

c) Define relatively prime integers.

d) Prepare composition table for $(Z_5, \times_5)$.

e) Find real and imaginary part of Z = 1 – i

f) Find the Hamming distance between x = 00000 and y = 11111

P.T.O.

Q.2 Attempt **ANY THREE** of the following **(12)**

a) Prove that $2^{n+1} \leq 2^{n}$ for $n \geq 3$ by using first principle of mathematical induction.

b) Prove that if (a, m) = (b, m) = 1 then (ab, m) = 1.

c) Prove that $\left(1 + i\sqrt{3}\right)^{-10} = 2^{-11}(-1 + i\sqrt{3})$.

d) Find all code words of the code determined by the parity check matrix

$$H = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Q.3 Attempt **ANY FOUR** of the following: **(12)**

a) Check whether the function $f(x) = x^3 - x$ is one – one or not

b) Draw the directed graph of relation R on a set A = {a, b, c, d, e } is defined as R ={(a, a), (a, b), (b, c), (b, d), (c, d), (c, e), (d, b), (d, c), (e, a)}

c) Prove that let x, y, z ∈ B^m then
1) δ (x, y) = δ(y, x)
2) δ (x, y) ≥ 0

d) Find the modulus and argument of $z = \frac{i - i^2 + i^3 + i^4}{i^5 + i^6 + 2}$

e) Obtain gof if $f : R \rightarrow R$ such that $f(x) = x^2 - 1$ and $g : R \rightarrow R$ such that $g(x) = \frac{3x-4}{10}$.

Q.4 Attempt **ANY TWO** of the following **(12)**

a) Prove that R is equivalence relation, let R be a relation on Z define by xRy if and only if 5x + 6y is divisible by 11, for $x, y \in Z$

b) Find the matrix of transitive closure for

$$M(R) = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ using Warshall's algorithm.

c) State De – Moivre's theorem and prove that any two cases.

Q.5 Attempt **ANY TWO** of the following: **(12)**

a) Find the g.c.d. of 3927 and 377 and express the g.c.d. in the form 3927m + 377n

b) If a, b, c, d ∈ Z, n ∈ N and $a \equiv b(mod\ n), c \equiv d(mod\ n)$

i. $(a + c) \equiv (b + d)(mod\ n)$

ii. $(a - c) \equiv (b - d)\ (mod\ n)$

iii. $ac \equiv bd\ (mod\ n)$

c) Construct a decoding table with syndroms for a group code given by generator matrix $G = \begin{bmatrix} 1\,0\,1\,1\,0 \\ 0\,1\,0\,1\,1 \end{bmatrix}$. Use the table to decode the received word 11110.

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