F.Y.B.SC. (COMPUTER SCIENCE) SEM –II (2014 COURSE) :
WINTER - 2017
SUBJECT : ALGEBRA – II

Day : **Wednesday** Time : **03.00 PM TO 05.00 PM**
Date : **01/11/2017** Max. Marks : 40

W-2017-0739

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Prove that intersection of two subgroups of a group is a subgroup and check weather the union of two subgroups is subgroup.

b) Show that $(Z_5, +)$ is a cyclic group find all its generator's.

c) Prove that $(Q_1, *)$ is a group, if $Q = Q_1 - \{1\}$ and define the composition * on Q_1 as $a * b = a + b - ab\ \forall\ a, b \in Q_1$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ in S_8 as a product of disjoint cycles. Determine whether σ is even or odd find σ^{-1}.

b) Prove that :

i) If G is a group then the identity element of G is unique.

ii) If a is any element in a group G then $(a^{-1})^{-1} = a$.

c) Show that A_3 is normal S_3.

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) Prove that a subgroup H of group G is normal if and only if $xHx^{-1} = H, \forall\ x \in G$.

b) Show that (Z, +) is isomorphic to (mZ, +) if $f : Z \to mZ$ define as $f(n) = mn$.

c) Find the quotient group $\frac{Z_2 \times Z_3}{H}$ where $H = \{(0, 0), (0, 1), (0, 2)\}$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) Prepare composition table for addition and multiplication for Z_4.

b) Define any two properties of group.

c) Given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$

Find: **i)** fog **ii)** $f^{-1}og^{-1}$

d) Define Kernel of Homomorphism.

e) State true or false: $O(S_n) = (n - 1)!$, where S_n is set of all permutation on n symbols.

f) Define Ring.

g) Find all solutions of the equation: $x^2 + 2x + 2 = 0$ in Z_6.

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