F.Y. B. SC. (COMPUTER SCIENCE) SEM –II (CBCS - 2016 COURSE) : WINTER - 2017

SUBJECT : ALGEBRA – II

Day : Monday Time : 03.00 PM TO 06.00 PM

Date : 30/10/2017 Max. Marks : 60

W-2017-0712

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of statistical tables and **CALCULATOR** is allowed.

Q.1 A) Select the correct option and rewrite complete sentence: **[06]**

i) Inverse of i in a group $G = \{1, -1, i, -i\}$ with respect to multiplication is _____.

a) 1 **b)** –1 **c)** i **d)** –i

ii) The order of an element $\bar{3}$ in $(Z_6, +_6)$ is ______.

a) 1 **b)** 2 **c)** 3 **d)** 6

iii) Number of subgroups of cyclic group G, of order 12 is _________.

a) 3 **b)** 6 **c)** 9 **d)** 12

iv) The number of generators of cyclic group $(Z_5, +_5)$ is ________.

a) 4 **b)** 8 **c)** 12 **d)** 16

v) The order of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 2 & 5 & 4 & 1 \end{pmatrix}$ is _______.

a) 1 **b)** 2 **c)** 3 **d)** 4

vi) The Kernel of $f(x) = 2x, \forall x \in Z$ is ________.

a) Kerf = {2} **b)** Kerf = {x} **c)** Kerf = {0} **d)** Kerf = {1}

B) Attempt the all following: **[06]**

i) Is union of two subgroups is subgroup. Justify with example.

ii) Find order of σ, if $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 1 & 6 & 5 & 8 & 7 \end{pmatrix}$.

iii) Find ϕ (36).

iv) Find the order of each element in $(Z_3, +_3)$.

v) State true or false : $O(S_n) = (n - 1)\,!$, when S_n is set of all permutations on n symbols.

vi) Define field.

P.T.O.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Find all generators of cyclic group $(Z_5, +_5)$.

b) Show that the multiplicative group G = $\{1, \omega, \omega^2\}$ where ω is complex cube root of unity, is isomorphic to $(Z_3, +_3)$.

c) Express the permutation $g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$ as a product of disjoint cycles. Determine whether g is even or odd. Also find g^{-1}.

d) Find the quotient group of $\frac{Z_2 \times Z_3}{H}$, where H = {(0, 0), (0, 1), (0, 2)}.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Show that f is homomorphism and find ker (f) if $f : (Z, +) \rightarrow (Z_n, +_n)$ be a function defined by $f(a) = \bar{a}$.

b) Prove that every subgroup of abelian group is normal.

c) Find all subgroups of $(Z_8, +_8)$.

d) Prepare composition table for $\left(Z_7^*, \times_7\right)$.

e) Find all solutions of $x^3 - 2x^2 - 3x = 0$ in Z_{12}.

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Prove that the intersection of two subgroups of a group is a subgroup.

b) State and prove left cancellation law and right cancelation law.

c) Compute: **i)** fog **ii)** $h^{-1}og^{-1}$ **iii)** $(fog)oh$

If given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, $h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Prove that if H is subgroup of a group G, and G is normal if and only if $xHx^{-1} = H, \forall\ x \in G$.

b) Show that A_3 is normal in S_3.

c) Show that $(Q^+, *)$ is a group where $a * b = \frac{ab}{2}$ for a, b $\in Q^+$.

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