B.Tech Sem - III (2007 Course) (Computer Eng g.) /Electrical Engg / Electronic Engg./Inf. Tech./Biomedical Engg./ E & TC Engg.) : WINTER - 2018

SUBJECT: ENGINEERING MATHEMATICS-III

Day : Friday
Date : 23/11/2018

W-2018-2705

Time : 10.00 AM TO 01.00 PM
Max. Marks: 80

N. B. :

1) **Q. No.1** and **Q. No.5** are **COMPULSORY.** Out of the remaining attempt **ANY TWO** questions from Section-**I** and **ANY TWO** questions from Section-**II**.
2) Figures to the right indicate **FULL** marks.
3) Answer to the both sections should be written in **SEPARATE** answer book.
4) Assume suitable data, if necessary.
5) Use of non-programmable **CALCULATOR** is allowed.
6) Draw neat and labeled diagram **WHEREVER** necessary.

SECTION-I

Q.1

a) Solve: $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\left[\log(1+x)\right]$. **(05)**

b) Find the Fourier cosine transform of $f(x)=2e^{-5x}+5e^{-2x}$. **(04)**

c) If $u=\frac{1}{2}\log(x^2+y^2)$, find V such that $f(z)=u+iv$ is analytic. Determine f (z) in terms of z. **(05)**

Q.2 Solve (**ANY THREE**): **(13)**

a) $(D^2+2D+1)y=4\sin 2x$.

b) $(D^2+4)y=x\sin x$.

c) $(D^4-m^4)y=\sin mx$. .

d) $(D^2+4)y=\tan 2x$ (By method of variation of parameters).

Q.3

a) Evaluate $\oint_C \frac{1}{z^2}=dz$, Where C is the circle $|z|=1$. **(04)**

b) Evaluate $\int_0^{2\pi}\frac{d\theta}{(5-3\cos\theta)^2}$. **(05)**

c) Show that, under the transformation $w=\frac{i-z}{i+z}$, x-axis in z-plane is mapped onto the circle $|z|=1$. **(04)**

Q.4

a) Find the Fourier sine transform of the function $f(x)=e^{-x}$ and hence show that $\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi}{2}e^{-m}$. **(05)**

b) Find the Fourier cosine integral representation for the function, **(04)**

$$f(x)=\begin{cases}1, & 0\le x\le 1.\\ 0, & x>1\end{cases}$$

c) Find $Z\{f(k)\}$ *where* $f(k)=\cos\left(\frac{k\pi}{4}+\alpha\right)$, $k\ge 0$. **(04)**

P.T.O.

SECTION-II

Q.5 **a)** Find the Laplace transform of the following: **(06)**
i) $\sin^2 t$ ii) cos2t cos4t

b) Find the work done by, $\vec{F} = 2xy^2\,\vec{i} + (2x^2 y + y)\,\vec{j}$ in taking a particle from (0,0,0) to (2,4,0) along the parabola $y = x^2$, $z = 0$. **(04)**

c) If $\vec{r} \times \dfrac{\overrightarrow{dr}}{dt} = 0$, show that $\vec{r}$ has a constant direction. **(04)**

Q.6 **a)** Find the inverse Laplace transform of $\dfrac{2s+5}{(s+1)(s-2)}$. **(04)**

b) Find the inverse Laplace transform of $\log\left(\dfrac{s+1}{s+2}\right)$. **(04)**

c) Solve the differential equation **(05)**
$\dfrac{d^2y}{dt^2} + 2\dfrac{dy}{dt} + y = te^{-t},\ y(0) = 1,\ y'(0) = -2.$

Q.7 **a)** Prove that $\vec{F} = (2xy + z^3)\vec{i} + x^2\,\vec{j} + 3xz^2\,\vec{k}$ is irrotational force field. Hence find corresponding scalar potential. **(05)**

b) Show that $\nabla^4(\log r) = \dfrac{2}{r^4}$. **(04)**

c) Find the directional derivative of the function $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ is the direction of PQ where Q is (5, 0, 4). **(04)**

Q.8 **a)** Evaluate $\iint_S \vec{F}.\overrightarrow{ds}$ where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ *and* S is the part of the surface of the sphere $x^2 + y^2 + z^2 = 1$ which lies in the first octant. **(06)**

b) Verify stoke's theorem for $\vec{F} = xy^2\,\vec{i} + y\,\vec{j} + z^2x\,\vec{k}$ for the surface of rectangular lamina bounded by x = 0, y = 0, x = 1, y = 2, z = 0. **(07)**

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