

**B.Tech. SEM -I (Chemical/ Civil/ Electrical/ Mechanical/ Production/  
Computer/ Info. Tech./ Electronics / Bio Medical / E & TC) 2014  
Course (CBCS) : WINTER - 2018  
SUBJECT : ENGINEERING MATHEMATICS – I**

Day : Thursday  
Date : 22/11/2018

Time : 10.00 AM TO 01.00 PM  
Max. Marks : 60

**W-2018-2260**

**N. B. :**

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable calculator is **ALLOWED**.
- 4) Assume suitable data, if necessary.

**Q.1 a)** Reduce the following matrix to its normal form hence find its rank: **(05)**

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

**b)** Examine for consistency and if consistent then solve it. **(05)**

$$\begin{aligned} x + y + z &= 6 \\ 2x + y + 3z &= 13 \\ 5x + 2y + z &= 12 \\ 2x - 3y - 2z &= -10 \end{aligned}$$

**OR**

Find eigen values and eigen vectors for the following matrix: **(10)**

$$\begin{bmatrix} 2 & -2 & 3 \\ 1 & 4 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

**Q.2 a)** If  $P$  is real and complex number  $\frac{1+i}{2+ip} + \frac{2+3i}{3+i}$  is represented on **(05)**

Aryand's diagram by a point on a line  $y = x$ , show that  $P = -5 \pm \sqrt{21}$

**b)** Two opposite vertices of a square are represented by complex numbers **(05)**  
 $(9 + 2i)$  and  $(-5 + 10i)$ . Find the complex number representing the other  
two vertices of the square.

**OR**

**a)** If  $\tan hx = \frac{2}{3}$ , find the value of  $\cos h 2x$ . **(05)**

**b)** If  $\tan\left(\frac{x}{2}\right) = \tanh\left(\frac{u}{2}\right)$ . Prove that  $\sinh x = \tan x$  **(05)**

**P. T. O.**

**Q. 3 a)** Find  $n^{\text{th}}$  derivatives of:  $y = \frac{x^3}{(x-1)(x-2)}$  (05)

**b)** Prove that: (05)

$$e^x \cos x = 1 + x - \frac{x^3}{3} - \frac{x^4}{6} + \dots$$

**OR**

If  $y = (\sin^{-1} x)^2$ , Prove that  $(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - n^2 y_n = 0$ . (10)

**Q. 4 a)** Evaluate:  $\lim_{x \rightarrow 1} \frac{\sqrt{1+x} - \sqrt{1-x^2}}{\sqrt{1-x} - \sqrt{1-x^2}}$  (05)

**b)** Find a and b if  $\lim_{x \rightarrow 0} \frac{a \sinh x + b \sin x}{x^3} = \frac{5}{3}$  (05)

**OR**

**a)** Test for convergence the series whose  $n^{\text{th}}$  term is  $\sqrt{\frac{2^{n-1}}{3^{n-1}}}$  (05)

**b)** Test for convergence the series  $\sum_{n=0}^{\infty} \frac{1}{n^2 + 1}$  (05)

**Q. 5 a)** If  $u = \tan(y + ax) - (y - ax)^{3/2}$  then verify  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$ . (05)

**b)** If  $x^3 + y^3 = 3ax^2$  then find  $\frac{\partial^2 y}{\partial x^2}$  (05)

**OR**

Find:

$$x^2 W_{xx} + 2xy W_{xy} + y^2 W_{yy} \text{ if } W = \operatorname{cosec}^{-1} \sqrt{\frac{\sqrt{x} + \sqrt{y}}{\sqrt[3]{x} + \sqrt[3]{y}}} + \tan^{-1} \left( \frac{x^3 + y^3}{2x + 3y} \right) \quad (10)$$

**Q. 6** Discuss the maxima and minima of the following function: (10)

$$x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$$

**OR**

**a)** If  $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$  (05)

Find:  $\frac{\partial x}{\partial u}$

**b)** Examine for functional dependence of  $u = \frac{x-y}{x+y}, v = \frac{x+y}{x}$ , (05)

if functionally dependent find the relation between them.