S.Y.B.SC. SEM – IV (2014 Course) : WINTER - 2018
SUBJECT: MATHEMATICA: VECTOR CALCULUS

Day: Monday
Date: 22/10/2018

W-2018-0820

Time: 03.00 PM TO 05.00 PM
Max. Marks: 40

N.B:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(10)**

a) Prove that a differentiable vector function $\overline{u}(t)$ on [a,b] is of constant magnitude if and only if $\overline{u}\cdot\frac{d\overline{u}}{dt}=0,\ \forall t\in[\text{a, b}]$.

b) If $\overline{r}=\frac{a}{2}(x+y)\,\overline{i}+\frac{b}{2}(x-y)\,\overline{j}+xy\,\overline{k}$, then find

i) $\left[\frac{\partial\overline{r}}{\partial x}\ \frac{\partial\overline{r}}{\partial y}\ \frac{\partial^2\overline{r}}{\partial x^2}\right]$ ii) $\left[\frac{\partial\overline{r}}{\partial x}\ \frac{\partial\overline{r}}{\partial y}\ \frac{\partial^2\overline{r}}{\partial x\,\partial y}\right]$

c) Find the scalar function $\phi(x,y,z)$ if
$\text{grad}\,\phi=y(2xz-1)\,\overline{i}+x(xz-1)\,\overline{i}+(x^2y+4)\,\overline{k}$ and ϕ (2,1,-1) = -10.

Q.2 Attempt **ANY TWO** of the following: **(10)**

a) If $\overline{u}$ is vector point function and ϕ is a scalar point function then show that
$\nabla\times(\phi\overline{u})=(\nabla\phi)\times\overline{u}+\phi(\nabla\times\overline{u})$

b) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of $\overline{i}+2\overline{j}+2\overline{k}$

c) Evaluate $\iint_S \overline{f}\cdot\overline{n}\,ds$ if $\overline{f}=z\overline{i}+x\overline{j}-3y^2z\overline{k}$ and S is the surface of the cylinder $x^2 + y^2 = 16$ included in first octant between z = 0 and z = 5.

Q.3 Attempt **ANY TWO** of the following: **(10)**

a) State and prove Green's theorem in the plane.

b) Evaluate $\iint_S\left[(x+z)\,dy\,dz+(y+z)\,dz\,dx+(x+y)\,dx\,dy\right]$
Using divergence theorem, where S is the surface of sphere $x^2 + y^2 + z^2 = 4$.

c) Using Stoke's theorem evaluate $\int\overline{f}\cdot d\overline{r}$, where $\overline{f}=4xz\overline{i}-y^2\overline{j}+yz\overline{k}$ over the area in the plane z = 0 bounded by x = 0, y = 0 and $x^2 + y^2 = 1$.

P.T.O.

Q.4 Attempt **ANY FIVE** of the following: **(10)**

a) Prove that $\nabla r^n = nr^{n-2}$ *where* $\bar{r} = x\bar{i} + y\bar{i} + \bar{z}$.

b) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$, $|\bar{r}| = r$ *then find* $\nabla \times \bar{r}$.

c) Find maximum value of directional derivative of $\phi = xy^2 + yz^3$ at the point P (2, -1, 1).

d) If $\bar{f}(t) = (t - t^2)\bar{i} + 2t^3\bar{j} - 3\bar{k}$, *find* $\int_1^2 \bar{f}(t)\,dt$.

e) Eliminate $\bar{a}$ *and* $\bar{b}$ *from* $\bar{r} = \bar{a}\cos 2t + \bar{b}\sin 2t$ and obtain the differential equation.

f) State Gauss's Divergence theorem.

g) Evaluate $\int \bar{f} \cdot d\bar{r}$, *where* $\bar{f} = y^2\bar{i} + 2xy\bar{j}$ from O (0,0) to P (1,1) along the straight line OP.

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