

Day : Wednesday
Date : 24/10/2018

W-2018-0808

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

N.B.:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

- a) Let G be a group. Let $a, b \in G$. prove that $(b^{-1}ab)^m = b^{-1}a^mb$, where $m \in \mathbb{N}$.
- b) Show that the set of four matrices $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ forms a group under matrix multiplication. Is it abelian? Why?
- c) Solve the differential equation $xp^2 - 2yp + ax = 0$, where $p = \frac{dy}{dx}$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

- a) Show that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1} \in H, \forall a, b \in H$.
- b) Find all the subgroups of a cyclic group of order 36.
- c) Solve the differential equation $y = 2px - p^2$, where $p = \frac{dy}{dx}$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

- a) Show that for the equation $f(D)y = X$, when $X = e^{ax}$, P.I. is $\frac{1}{f(D)}e^{ax} = \frac{e^{ax}}{f(a)}$, if $f(a) \neq 0$.
- b) Solve : $(D^2 - 6D + 13)y = e^{3x}(2\sin x \cdot \cos x)$.
- c) Solve : $(D^4 + 2D^3 - 3D^2)y = x^2 + 3e^{2x}$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

- a) Solve : $(D^3 + 1)y = 0$.
- b) Solve : $p^2 - 7p + 12 = 0$, where $p = \frac{dy}{dx}$.
- c) Find particular integral of $(D^4 + 3D^2 - 4)y = \cos 3x$.
- d) Show that the substitutions $x^2 = u$ and $y^2 = v$, convert the differential equation of the type $y^2 = pxy + f\left(\frac{py}{x}\right)$ into Clairaut's form.
- e) Define cyclic subgroup.
- f) Prove that in any group identity element is unique.
- g) Consider the group $(S, *)$, where $s = \{1, \omega, \omega^2\}$ and $*$ is usual multiplication of complex numbers and ω is a complex cube root of unity. Find inverse of every element of S .

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