S.Y.B.SC. SEM – IV (CBCS - 2016 Course) : WINTER - 2018

SUBJECT: MATHEMATICS : VECTOR CALCULUS

Day: Monday | **W-2018-0731** | Time: 03.00 P.M. To 06.00 P.M

Date: 22/10/2018 | Max. Marks: 60

N.B:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) A non-constant vector function $\bar{u}$ (t) is of constant direction if and only if $\bar{u} \times \frac{d\bar{u}}{dt} = \bar{0}$.

b) If $\bar{a}=t^2\bar{i} + t\bar{j} + (2t+1)\bar{k}$ and $\bar{b} = (2t-3)\bar{i}+\bar{j}-t\bar{k}$,

Find: $i)\ \frac{d}{dt}(\bar{a}.\bar{b})$ $\quad ii)\ \frac{d}{dt}(\bar{a}\times\bar{b})$ $\quad iii)\ \frac{d}{dt}\left(\bar{a}\times\frac{d\bar{b}}{dt}\right)$.

c) If $\bar{r}$ $a\cos t\ \bar{i} + a\sin t\ \bar{j} + at\ \tan\alpha\ \bar{k}$,

Find: $i)\ \left|\frac{d\bar{r}}{dt}\times\frac{d^2\bar{r}}{dt^2}\right|$ $\quad ii)\ \left[\frac{d\bar{r}}{dt}, \frac{d^2\bar{r}}{dt^2}, \frac{d^3\bar{r}}{dt^3}\right]$.

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) If $\bar{u} = x^2yz\,\bar{i} - 2xz^3\,\bar{j} + xz^2\,\bar{k}$ and $\bar{v}=2z\bar{i} + y\bar{j} - x^2\bar{k}$ find $\frac{\partial^2}{\partial x\,\partial y}(\bar{u}\times\bar{v})$ at (1, 0, -2).

b) If $\bar{f} = (uvw)\bar{i} + uw^2\,\bar{j} - v^3\bar{k}$ and $\bar{g} = u^3\,\bar{i} - uvw\bar{j} + u^2\,w\bar{k}$

then find $\frac{\partial^2\bar{f}}{\partial v^2}\times\frac{\partial^2\bar{g}}{\partial u^2}$ at $(1,1,-1)$.

c) If $\bar{f} = (y+\sin z)\bar{i} + x\bar{j} + x\cos z\bar{k}$, show that $\bar{f}$ is irrotational and find ϕ such that $\nabla\phi = \bar{f}$.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) Find the directional derivative of $\phi = xy^2 + yz^3$ at the point P (2, −1,1) in the direction of $\bar{i}+2\bar{j}+2\bar{k}$. Find the magnitude of maximum directional derivative at P.

b) If $\bar{u}$ is a vector point function and ϕ is a scalar point function then show that $\nabla\cdot(\phi\bar{u}) = (\nabla\phi)\cdot\bar{u} + \phi(\nabla\cdot\bar{u})$.

c) If $\phi = 3x^2y,\ \psi = xz^2 - 2y$, evaluate $div\left(grad\left[(grad\,\phi).(grad\,\psi)\right]\right)$.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) The acceleration $\bar{a}$ of a particle at any time t is given by $\bar{a}=e^{-t}\bar{i}-6(t+1)\bar{j}+3\sin t\,\bar{k}$. If the velocity $\bar{v}$ and displacement $\bar{r}$ are zero at $t=0$, find $\bar{v}$ *and* $\bar{r}$ at any time.

b) Evaluate $\int_C \left[\left(x^2-y^2\right)\bar{i}+2xy\bar{j}\right].d\bar{r}$ around a rectangle with vertices at (0,0), (a,0), (a, b) and (0, b) traversed in counter – clockwise direction.

c) Evaluate $\int_C (\cos x\sin y)\,dx+(\sin x\cos y)\,dy$ by Green's theorem, where C is the unit circle with its centre at the origin.

d) Prove that $div\left(curl\,\bar{u}\right)=o.$ that is $\nabla\cdot\left(\nabla\times\bar{u}\right)=0.$

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Find the equation of tangent plane to the surface xyz = 4 at (1, 2, 2).

b) If $\bar{a}=t^2\bar{i}-(t+1)\bar{j}+t^3\bar{k}$ *and* $\bar{b}=2t\bar{i}+\bar{j}-\bar{k}$, *find* $\frac{d}{dt}(a+b)$.

c) Define curl of a vector point function and find $\nabla\times\bar{v}$ *where* $\bar{v}=x^2\bar{i}+y^2\bar{j}+z^2\bar{k}.$

d) If $\bar{v}=(x+y+1)\bar{i}+\bar{j}-(x+y)\bar{k}$, verify that $\bar{v}.curl\,\bar{v}=0.$

e) Let $\bar{f}(t)=t\bar{i}-t^2\bar{j}+(t-1)\bar{k}$ *and* $\bar{g}=2t^2\bar{i}+6t\bar{k}$, Evaluate $\int_0^2\left(\bar{f}\times\bar{g}\right)dt.$

f) If $\bar{f}=\left(3x^2+6y\right)\bar{i}-14yz\bar{j}+20xz^2\bar{k}$, evaluate $\int_C \bar{f}.dr$ from (0, 0, 0) to (1, 1, 1) along the path C which is straight line joining (0, 0, 0) and (1, 1, 1).

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