S.Y.B.SC. SEM – III (2014 Course) : WINTER - 2018

SUBJECT : MATHEMATICS: CALCULUS OF SERVERAL VARIABLES (M - 31)

Day : Monday
Date : 22/10/2018

W-2018-0806

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

N. B. ;

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q. 1 Attempt any **TWO** of the following: **(10)**

a) State and prove Euler's theorem on homogenous functions of two variables.

b) *If* $w = f(u, v)$, where $u = x^2 + y^2$, $v = 2xy$

then prove that $\left[x \dfrac{\partial w}{\partial x} - y \dfrac{\partial w}{\partial y} \right]^2 = 4\left(u^2 - v^2\right)\left(\dfrac{\partial w}{\partial u}\right)^2$,

given that w is a differentiable function of u and v.

c) Prove that $u = \dfrac{1}{\sqrt{x^2 + y^2 + z^2}}$, where $x^2 + y^2 + z^2 \neq 0$, satisfies the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2} = 0$.

Q. 2 Attempt any **TWO** of the following: **(10)**

a) State and prove Taylor's theorem for a function of two variables x and y.

b) Expand $x^2 + 2xy + yz + z^2$ about $(1, 1, 0)$, by using Taylor's theorem.

c) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$.

Q. 3 Attempt any **TWO** of the following: **(10)**

a) Change the order of integration and hence evaluate $\displaystyle\int_0^1 \left[\int_y^1 e^{-x^2}\, dx \right] dy$.

b) Find the volume of the ellipsoid by triple integration.

c) Evaluate $\displaystyle\iint_R \frac{dx\, dy}{xy}$, where R is the region in the xy-plane bounded by four circles $x^2 + y^2 = ax$, $x^2 + y^2 = bx$, $x^2 + y^2 = cy$ and $x^2 + y^2 = dy$ by using the transformations $\dfrac{x^2 + y^2}{x} = u$ and $\dfrac{x^2 + y^2}{y} = v$.

P. T. O.

Q. 4 Attempt any **FIVE** of the following: **(10)**

a) Can $f(0, 0)$ be defined so that $f(x, y)$ is continuous at $(0, 0)$ if

$$f(x, y) = \frac{\sin(x^2 + y)}{x + y}.$$

b) Define limit of function of two variables along a path.

c) If $u = e^{-x/y}$, then find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

d) If $u = \sin^{-1}\left[\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}\right]^{1/2}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$.

e) Evaluate $\frac{\partial(F,G)}{\partial(x,y)}$, if $F(x, y) = 3x^2 - xy$ and $G(x,y) = 2xy^2 + y^3$.

f) Evaluate $\int_0^1\int_0^1 \frac{dx\,dy}{\sqrt{(1-x^2)(1-y^2)}}$.

g) Change the order of integration of $\int_{-1}^{1}\left[\int_0^{\sqrt{1-x^2}} f\,dy\right]dx$.

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