F.Y.B.SC. SEM – I (CBCS - 2016 Course) : WINTER - 2018

SUBJECT : MATHEMATICS : CALCULUS

Day : Saturday
Date : 27/10/2018

W-2018-0692

Time : 11.00 A.M TO 02.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 A) Choose the correct alternatives of the following: **[06]**

i) $\lim_{x\to 0} x\tan\left(\frac{\pi}{2}-x\right) =$ ________.

a) 0 **b)** 1 **c)** $\frac{1}{2}$ **d)** $-\frac{1}{2}$

ii) $1+x-\frac{x^3}{3}-\frac{x^4}{6}-\frac{x^5}{30}+.........$ $\forall x \in R$ is an expansion of ________.

a) $e^x \cos x$ **b)** $\frac{1}{x}$ **c)** $\cos x$ **d)** e^x

iii) A series $\sum_{n=1}^{\infty} \frac{n}{2^n}$ is ___________.

a) divergent **b)** oscillatory **c)** convergent **d)** none of these

iv) If $y = a^{mx}$ then $y_n =$ ________.

a) $m^{n+1} (\log a)^n a^{mx}$
b) $m^{n-1} (\log a)^n a^{mx}$
c) $m^n (\log a)^n a^{mx}$
d) None of these

v) A sequence $\{a_n\}$ where $a_n = 2(-1)^n - \frac{3}{n}$ is ________.

a) convergent **b)** divergent **c)** oscillatory **d)** none of these

vi) If $y = \frac{1}{5+7x}$ then $y_n =$ _____________.

a) $\frac{(-1)^{n+1} n!(7)^n}{(5+7x)^n}$
b) $\frac{(-1)^n n!(7)^{n+1}}{(5+7x)^n}$
c) $\frac{(-1)^n n!(7)^n}{(5+7x)^{n+1}}$
d) $\frac{(-1)^n n!(7)^n}{(5+7x)^n}$

B) Answer the following: **[06]**

i) State geometrical meaning of Lagrange's mean value theorem.

ii) If $y = \sin^3 x$ then find y_n.

iii) Evaluate : $\lim_{x\to 0}\left[\frac{1}{x}-\frac{1}{e^x-1}\right]$.

iv) Define oscillation of a function.

v) State the comparison test.

vi) If $y = 5^{7x} + e^{2x} + \frac{1}{3x+4}$, find y_n.

P.T.O.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) State and prove Rolle's mean value theorem.

b) Verify Cauchy's mean value theorem for the functions $f(x)=\frac{1}{x^2}$ and $g(x)=\frac{1}{x}$ in [a, b], a > 0. Show that point 'C' is harmonic mean of a and b.

c) Using Lagrange's mean value theorem prove that,
$\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$.

d) Show that $\{a_n\}$ where $a_n=\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\frac{1}{(n+3)^2}+.......+\frac{1}{(n+n)^2}$ is bounded.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Discuss the continuity of function $f(x)=\frac{1}{1-e^{\frac{1}{x}}}$ where $x \neq 0$ and $f(0) = 0$.

b) Show that every continuous function on closed and bounded interval attains its bounds.

c) Evaluate : $\lim_{x\to 0}\frac{x-\log(1+x)}{x^2}$.

d) Show that the function f defined by $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

e) Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ on $\left[-\sqrt{2}, \sqrt{2}\right]$.

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if $p > 1$.

b) Using Maclaurin's theorem, prove that $\log\sec x=\frac{x^2}{2!}+\frac{2x^4}{4!}+\frac{16x^6}{6!}+.....$

c) Discuss the convergence of the series $\sum_{n=1}^{\infty}\frac{n+1}{n^3+2n^2+5}$ by using comparison test.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) State and prove Leibnitz's theorem for n^{th} derivative of the product of two functions of x.

b) If $y - \sin^{-1}x$, then show that $(1-x^2)\, y_2 - xy_1 = 0$. Hence deduce that
$(1-x^2)\, y_{n+2} - (2n+1)\, xy_{n+1} - n^2 y_n = 0$.

c) Show that a sequence $\{S_n\}$ where $S_n=\left(1+\frac{1}{n}\right)^n$ is monotonic and bounded.

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