F.Y.B.Sc. SEM – I (CBCS 2018 COURSE) : WINTER - 2018

SUBJECT : MATHEMATICS : ALGEBRA

Day : Tuesday
Date : 23/10/2018

W-2018-0675

Time : 11.00 A.M TO 02.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Prove that if A is a square matrix of order n, then the matrices A and adj. A. commute and the product is the scalar matrix |A| I .
i.e, A(adj A) = (adjA) A = |A| I.

b) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

c) Find the non-singular matrices P and Q such that PAQ is the normal form and find rank of A, where $A = \begin{bmatrix} 2 & 1 & 3 & -2 \\ 3 & -1 & 0 & 4 \\ 1 & 5 & 9 & 14 \end{bmatrix}$.

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Prove that any two non-zero integers a and b have an unique (positive) g.c.d. d = (a, b) and can be expressed in the form, d = (a, b) = ma + nb, for some m, n $\in \mathbb{Z}$.

b) Show that the integers 1357 and 1166 are relatively prime. Find integers m and n such that, 1 = 1357m + 1166n.

c) Solve the equation $x^4 + 1 = 0$ by using De Moivre's theorem.

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) State and prove De Moivre's theorem for positive and negative integers.

b) If Z_1 and Z_2 are any two complex numbers then show that :

i) $|z_1 z_2| = |z_1| |z_2|$

ii) $\arg(z_1 z_2) = \arg z_1 + \arg z_2$

c) Show that congruence relation in $\mathbb{Z}$ is an equivalence relation.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$.

b) Solve the following system of linear equations:

$$x + y + z = 0$$
$$2x - y - 3z = 0$$
$$3z - 5y + 4z = 0$$
$$x + 17y + 4z = 0$$

c) Express $\cos^7\theta$ in terms of the cosines of multiple angles.

d) If (a, m) = 1 = (b, m) then show that (ab, m) = 1.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Find the value of $\left(1+i\sqrt{3}\right)^{10} + \left(1+i\sqrt{3}\right)^{10}$.

b) Find the modulus and argument of $z = \frac{3-i}{2+i} + \frac{3+i}{2-i}$.

c) If $A = \begin{bmatrix} 4 & -1 & 3 \\ 0 & 1 & 2 \\ 2 & 1 & 3 \end{bmatrix}$, find A^{-1}.

d) Prove that if a | b and c | d then ac | bd.

e) Explain how to find the solution of homogenous system of linear equations of the form AX = 0.

f) Define : **i)** rank of a matrix.
ii) congruence modulo n.

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