S.Y.B.SC. SEM – IV (CBCS - 2016 Course) : WINTER - 2018

SUBJECT: MATHEMATICS: COMPLEX VARIABLES

Day : Wednesday		**W-2018-0733**	Time: 03.00 P.M. To 06.00 P.M
Date : 24/10/2018			Max. Marks: 60.

N.B.:

1) All questions are **COMPULSORY.**
2) Figures to the **RIGHT** indicate full marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Show that the real and the imaginary parts of an analytic function f(z) = u + iv, satisfy Laplace's differential equation.

b) If f(z) is an analytic function, then prove that

$$\nabla^2\left[Rf(z)\right]^2 = 2\left|f'(z)\right|^2 .$$

c) Evaluate: $\int_C \frac{e^z}{(z+1)^2}dz,$ where C is the circle |z–1|=3.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Evaluate: $\int_C \frac{e^z}{z}dz,$ where C is the circle |z|=1 and hence show that:

$$\int_0^{2\pi} e^{\cos\theta}\cos(\sin\theta)d\theta = 2\pi .$$

b) Obtain the expansion of $\frac{1}{(z^2+1)(z+2)}$ for $1<|z|<2$.

c) Evaluate by contour integration $\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$, where C is the circle | z–2 | = 2.

Q.3 Attempt any **TWO** of the following: **(12)**

a) If f(z) has a simple pole at z = z_0, then the residue of f(z) at z = z_0 is $\lim_{z\to z_0}(z-z_0)f(z)$.

b) Evaluate by contour integration $\int_{-\infty}^{\infty}\frac{dx}{x^2+1}$.

c) Prove that an analytic function with constant modulus is constant.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Find the analytic function whose real part is x^3-3xy^2.

b) Verify Cauchy-Goursat theorem for f(z) = z + 2 taken around the unit circle | z | = 1.

c) Evaluate $\int_0^{1+i}\left(x-y+ix^2\right)dz$ along the real axis from z = 0 to z = 1 and then along a line parallel to imaginary axis from z = 1 to z = 1 + i.

d) Find the residue of $f(z)=\frac{1}{z^2(z-i)}$ at z = i by expanding f(z) as a Laurent's series about z = i.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Show that the function $f(z)=\bar{z}$ continuous everywhere but differentiable no where.

b) Find f(1 + i) ; if f is continuous at z = 1 + i where $f(z)=\frac{z^4+4}{z-(1+i)}$, if $z \neq 1 + i$.

c) Find the zeros of $z^3 - 6z^2 + 11z - 6$.

d) Determine the poles and their orders for the function

$$f(z)=\frac{1}{(z-3)^4(z-2)^5}$$

e) Find the residue of $f(z)=\frac{z+2}{z^2-1}$ at $z=-1$.

f) Define: (i) Simple closed curve
(ii) Simply connected region

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