F.Y.B.SC. SEM – I (2014 Course) : WINTER - 2018
SUBJECT : MATHEMATICS : CALCULUS

Day : Thursday
Date : 25/10/2018

W-2018-0782

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Prove that every continuous function on closed and bounded interval attains its bounds.

b) Discuss the continuity of function f, if

$f(x)=\dfrac{x-1}{1+e^{\frac{1}{x-1}}}$ when $x \neq 1$ and $f(1)=0$.

c) If $y = (\sin^{-1}x)^2$, then prove that

$\left(1-x^2\right) y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) State and prove Lagrange's mean value theorem.

b) If $f(x)$ and $g(x)$ are continuous on [a, b] and derivable in (a, b) then prove that there exists c in (a, b) such that

$$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a) \begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix}$$

c) Verify Rolle's theorem for the function

$f(x)=\log\left[\dfrac{x^2+ab}{(a+b)x}\right]$, over $[a, b]$ *where* $a>0$

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) Show that $\sum_{n=1}^{\infty}\dfrac{1}{n^p}$ is convergent if p > 1.

b) Show that the following sequence is monotonic and bounded. Also find its limit.
0.5, 0.55, 0.555, 0.5555,---------.

c) Using Taylor's theorem expand sinx in ascending powers of $\left(x-\dfrac{\pi}{2}\right)$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) State Heine's property.

b) State Geometrical meaning of Rolle's mean value theorem.

c) Evaluate : $\lim_{x\to 0}\left[\dfrac{1}{x} - \dfrac{1}{\sin x}\right]$.

d) If $y = x^2 \sin 3x$, find y_n.

e) If $y = e^{5x} - \log(7x+3) + e^{3x} + 3^{2x}$, find y_n.

f) Discuss the convergence of $\{a_n\}$ where $a_n = 2(-1)^n - \dfrac{5}{n}$.

g) Discuss the convergence of $\sum \dfrac{2n^2+3}{3n^4+1}$.

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