

S.Y.B.SC. SEM – III (CBCS - 2016 Course) : WINTER - 2018

SUBJECT: MATHEMATICS: GROUP THEORY & DIFFERENTIAL EQUATIONS

Day: Friday
Date: 26/10/2018

W-2018-0719

Time: 11.00 A.M. To 02.00 P.M.
Max. Marks: 60

N.B:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: (12)

- a) If a, b are any two elements in a group G then show that $O(a) = O(b^{-1}ab)$.
- b) Let $G = \{ p, q, r, s \}$. In G an associative binary operation is given by the following table. Verify that G forms an abelian group.

*	p	q	r	s
p	p	q	r	s
q	q	p	s	r
r	r	s	p	q
s	s	r	q	p

- c) Prove that in any group:
 - i) There is an unique identity element.
 - ii) Every element has an unique inverse.

Q.2 Attempt any **TWO** of the following: (12)

- a) Prove that a non-empty subset H of a group G is a subgroup of G if and only if the following two conditions are satisfied:
 - i) $a, b \in H \Rightarrow ab \in H$
 - ii) $a \in H \Rightarrow a^{-1} \in H$
- b) Define cyclic group. Give an example of a cyclic group. Show that cyclic group is abelian.
- c) A and B are subgroups of a group G such that $A \cup B$ is also a subgroup of G . Show that $A \subseteq B$ or $B \subseteq A$.

Q.3 Attempt any **TWO** of the following: (12)

- a) Show that if $f(D)y = e^{ax}V$, where V is a function of x then

$$\frac{1}{f(D)}(e^{ax}V) = \frac{1}{f(D+a)}V.$$

P.T.O.

b) Solve : $(D^4 + 4) y = \cos 2x + \cos 4x$.

c) Solve : $(D^2 - 1) y = xe^{2x}$.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Show that the substitutions $x^2 = u$ and $y^2 = v$ converts equation $(px - y)(py + x) = 2p$ into Clairaut's equation and hence solve it.

b) Solve : $y = 2px - p^2$, where $p = \frac{dy}{dx}$

c) Solve : $p^3 - 4xyp + 8y^2 = 0$, where $p = \frac{dy}{dx}$

d) Solve: $e^{3x}(p - 1) + p^3 e^{2y} = 0$.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Show that $(Z_5, +)$ is cyclic group. Find all its generators.

b) Let $S = \{ 1, -1, i, -i \}$ and (S, \cdot) be a group where \cdot is multiplication of complex numbers. Find inverse of every element in S .

c) Find all the subgroups of a cyclic group of order 30.

d) Solve: $(D^6 + 6D^4 + 9D^2) y = 0$.

e) Find the particular integral of $(D^3 - 4D) y = \cos 3x$.

f) Solve: $p^2 - 7p + 12 = 0$, where $p = \frac{dy}{dx}$.

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