

F.Y.B.Sc. SEM – I (CBCS 2018 COURSE) : WINTER - 2018

SUBJECT : MATHEMATICS : CALCULUS

Day : Thursday
Date : 25/10/2018

W-2018-0677

Time : 11.00 A.M TO 02.00 PM
Max. marks : 60

N.B.

- 1) All questions are **COMPULSORY**.
- 2) Figures to the **RIGHT** indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. (12)

- a) Prove that every continuous function on closed and bounded interval is bounded.
- b) Discuss the continuity of a following function in the interval $[-2, 2]$, where
$$f(x) = 2 - 3x \quad , \quad \text{for } -2 \leq x \leq -1$$
$$= 2x + 7 \quad , \quad \text{for } -1 < x < 1$$
$$= 4x + 1 \quad , \quad \text{for } 1 \leq x \leq 2$$
- c) If $y = \sin(m \sin^{-1}x)$, then show that
$$(1 - x^2)y_{n+2} - (2n + 1)x y_{n+1} + (m^2 - n^2)y_n = 0$$

Q.2 Attempt **ANY TWO** of the following. (12)

- a) Show that a sequence $\{s_n\}$, where $s_n = \left(1 + \frac{1}{n}\right)^n$ is monotonic and bounded.
- b) Discuss the convergence of following series by using comparison test,
$$\sum \frac{2n^2 + 3n}{3n^3 - 4n + 5}$$
- c) By using Taylor's theorem, prove that
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$$

Q.3 Attempt **ANY TWO** of the following. (12)

- a) State and prove Lagrange's mean value theorem.
- b) Verify Rolle's theorem for the function $f(x) = 2x^3 + x^2 - 4x - 2$ on $[-\sqrt{2}, \sqrt{2}]$.
- c) Use Cauchy's mean value theorem to obtain value of 'c' for the functions $f(x) = e^x$ and $g(x) = x$ over $[0, 1]$.

Q.4 Attempt **ANY THREE** of the following. (12)

- a) Discuss the continuity of the function $f(x)$, if $f(x) = \frac{x-1}{1+e^{\frac{1}{x}-1}}$ when $x \neq 1$ and $f(1) = 0$.
- b) Evaluate : $\lim_{x \rightarrow 0} \frac{\log \tan x}{\log x}$.
- c) If $y = x^3 \sin 3x$ then find y_n .
- d) Show that a sequence $\{a_n\}$ whose n^{th} term is $a_n = \frac{n}{n+1}$ is monotonic increasing sequence.

P.T.O.

Q.5 Attempt **ANY FOUR** of the following.

(12)

a) Define : i) removable discontinuity
ii) irremovable discontinuity.

b) Discuss the continuity of $f(x) = \sqrt{(x-2)(x-4)}$.

c) Evaluate : $\lim_{x \rightarrow 0} x^x$.

d) If $y = (ax+b)^m$ then find y_n .

e) Discuss the convergence of series $\sum \frac{n!}{2^n}$

f) Define : i) bounded below
ii) bounded above.

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