F.Y.B.SC. SEM – I (CBCS - 2016 Course) : WINTER - 2018
SUBJECT: MATHEMATICS : ALGEBRA

Day: Thursday
Date: 25/10/2018

W-2018-0690

Time: 11.00 A.M TO 02.00 PM
Max. Marks: 60

N.B:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 A) Choose the correct alternative of the following **(06)**

i) Let X be set of all peoples of world. Let the relation R on X be defined as x R y iff x ia a friend of y________.
a) R is a reflexive relation b) R is a symmetric relation
c) R is a transitive relation d) none of these

ii) Product of complex conjugates is always________.
a) 0 b) a real number
c) an imaginary number d) none of these

iii) Least common multiple of any two non – zero integers is always ________.
a) unique positive integer b) a positive integer
c) can be a negative integer d) none of these

iv) Greatest common divisor of 15 and 18 is_______.
a) 3 b) 270
c) 90 d) none of these

v) Partition gives ___________.
a) a transitive relation only b) a symmetric relation
c) an equivalence relation d) none of these

vi) Congruence modulo n is a_________.
a) relation b) set
c) vector d) none of these

B) Attempt the following **(06)**

i) State the conditions for the consistency of system of non – homogeneous equations.
ii) Define the term 'eignevector'.
iii) If $-45 \equiv x \pmod 7$ then find the value of x.
iv) Give an example of a relation which is symmetric but neither reflexive nor transitive.
v) What is mean by ' Characteristic Equation'.
vi) Find the argument of a complex number $z = i - 3$

Q.2 Attempt any **THREE** of the following **(12)**

a) Find the value of $\dfrac{(\cos 8\theta + i\sin 8\theta)^{1/4}}{(\cos 3\theta - i\sin 3\theta)^{1/3}}$

b) Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$

c) In Z_7 find $\left(\bar{3}\right)^{-1} - \bar{5} = ?$

d) If a, b, c are integers such that a | b and a |c then show that a | (bx + cy) for all integers x, y.

P.T.O.

Q.3 Attempt any **FOUR** of the following **(12)**

a) Find the values of k so that the system of equations:
$x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + kz = 9$ have
i)No solution
ii)Unique solution.

b) If a, b, x, y be non – zero integers such that xa + yb = 1, then show that (a, b) = (x, y) = (x, b) = 1 .

c) If |z| = 1 and arg (z) = θ, show that $\frac{1+z}{1-z} = i\cot\left(\frac{\theta}{2}\right)$

d) Find the adjoint of a matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$

e) If p is a prime and a, b are integers such that p | ab then prove that either p | a or p | b.

Q.4 Attempt any **TWO** of the following **(12)**

a) Define 'Congruence modulo n'. Show that congruence modulo n is an equivalence relation.

b) Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$ and hence find its inverse.

c) Find the value of $\left(1+i\sqrt{3}\right)^{10} + \left(1-i\sqrt{3}\right)^{10}$.

Q.5 Attempt any **TWO** of the following: **(12)**

a) Find non – singular matrices P and Q such tht PAQ is the normal form of A hence determine the rank of A, where $A = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 14 & 20 \\ 1 & 2 & 3 \end{bmatrix}$

b) Define g.c.d. of two integers. Find g.c.d. of integers a = 3927, b = 377 .
Also find integers x, y such that d = ax + by .

c) Define modulus and argument of a complex number. If z_1 and z_2 are two complex numbers then prove that i) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and

ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2),\ z_2 \neq 0.$

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