F.Y.B.SC. SEM – I (2014 Course) : WINTER - 2018
SUBJECT: MATHEMATICS : ALGEBRA

Day : Tuesday
Date : 23/10/2018

W-2018-0780

Time : 12.00 NOON TO 02.00 PM
Max. Marks : 40

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Prove that a necessary and sufficient condition for a square matrix A to have the inverse is that A is non-singular matrix.

b) Determine the values of x(if any) that will make the matrix A given below of **(i)** rank 1 **(ii)** rank 2 **(iii)** rank 3, where

$$A=\begin{bmatrix} x & x & 2 \\ 2 & x & x \\ x & 2 & x \end{bmatrix}$$

c) Verify Cayley-Hamilton theorem for the matrix

$$A=\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

Q.2 Attempt any **TWO** of the following: **(10)**

a) Prove that any two non-zero integers a and b have an unique (positive) g.c.d and can be expressed in the form, $d=(a,b)=ma+nb$ for some integers m and n.

b) Find g.c.d. of 2210 and 357 and express the g.c.d. in the form 2210 m + 357 n. Find the values of m and n.

c) Test the consistency of following equations. If they are consistent find their general solution,
$2x+3y+3z+2u=8,\ \ 3x-2y-7z-5u=9,$
$5x+2y+4z-8u=5,\ \ x-y+5z-4u=-5$

Q.3 Attempt any **TWO** of the following: **(10)**

a) If z_1 and z_2 are any two complex numbers then show that **(i)** $|z_1z_2|=|z_1||z_2|$ and **(ii)** $\arg(z_1z_2)=\arg z_1+\arg .z_2$.

b) Show that $$\left[\frac{1+\sin\left(\frac{2\pi}{9}\right)+i\cos\left(\frac{2\pi}{9}\right)}{1+\sin\left(\frac{2\pi}{9}\right)-i\cos\left(\frac{2\pi}{9}\right)}\right]^{18}=-1$$

c) Solve the equation $x^4-x^3+x^2+1=0$, using De Moivre's theorem.

P.T.O.

Q.4 Attempt any **FIVE** of the following: **(10)**

a) Find the eigen values of the matrix

$$A = \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix}$$

b) Show that if A is a non-singular matrix $(A')^{-1} = (A^{-1})'$.

c) Find the modulus and argument of $z = \frac{3-i}{2+i} + \frac{3+i}{2-i}$.

d) Find the fourth roots of unity.

e) Find A^{-1}, if $A = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$.

f) Write the conditions when the homogeneous linear equations have **(i)** zero solution **(ii)** non-zero solution.

g) Define congruence relation with suitable example

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