S.Y.B.SC. SEM – III (CBCS - 2016 Course) : WINTER - 2018

SUBJECT: MATHEMATICS : CALCULUS OF SEVERAL VARIABLES

Day: Wednesday
Date: 24/10/2018

W-2018-0717

Time: 11.00 A.M. To 02.00 P.M.
Max. Marks: 60

N.B:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **(12)**

a) Show that if a function $f(x, y)$ is differentiable at (a, b), then (i) the partial derivatives $f_x(a,b)$ and $f_y(a, b)$ exist and (ii) f is continuous at (a,b).

b) Prove that $u = \frac{1}{\sqrt{x^2+y^2+z^2}}\left(x^2+y^2+z^2 \neq 0\right)$, satisfies the partial differential equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$.

c) If $x = r\cos\theta$, $y = r\sin\theta$, show that

$$\frac{\partial^2\theta}{\partial x\partial y}=\frac{\partial^2(\log r)}{\partial x^2}=-\frac{\partial^2(\log r)}{\partial y^2}=-\frac{1}{r^2}\cos 2\theta .$$

Q.2 Attempt **ANY TWO** of the following: **(12)**

a) Explain Lagrange's method of undetermined multipliers.

b) If $u=\sin^{-1}\left(x^2+y^2\right)^{\frac{1}{5}}$ then prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u\left[2\tan^2 u-3\right].$$

c) Find maxima or minima of $F(x,y) = x^4 + y^4 - (x+y)^2$.

Q.3 Attempt **ANY TWO** of the following: **(12)**

a) State and prove Taylor's theorem for a function of two variables.

b) By using Taylor's theorem, expand $f(x, y) = \sin xy$ in powers of $(x-1)$ and $\left[y-\frac{\pi}{2}\right]$ upto and including second degree terms.

c) Find the maximum value of $\phi(x, y, z) = xyz$ subject to the condition $\frac{x^2}{3}+\frac{y^2}{9}+\frac{z^2}{8}=1$.

P.T.O.

Q.4 Attempt **ANY THREE** of the following: **(12)**

a) Four parabolas whose equations are $y^2 = 4ax$, $y^2 = 4bx$, $x^2 = 4cy$, $x^2 = 4dy$ intersect and form a quadrilateral space. Find the area of the space thus enclosed.

b) Find by triple integration the volume of the solid bounded by $z = 0, x^2 + y^2 = 1$ and $x + y + z = 3$.

c) Change the order of integration and hence evaluate $\int_0^1 \left[\int_y^1 e^{-x^2}\, dx \right] dy$.

d) Evaluate $\iint_D (y-x)$ over the region D in the xy-plane bounded by the lines

$y=x+1, y=x-3,\ y=-\frac{1}{3}x+\frac{7}{3}, y=-\frac{1}{3}x+5.$

Q.5 Attempt **ANY FOUR** of the following: **(12)**

a) Examine the continuity of $f(x,y)$ at the origin, where

$f(x,y) = \frac{x+y}{x-y}$ for $x \neq y$ and $f(0,0)$.

b) *If* $u=\log\left(x^3+y^3-x^2y-xy^2\right)$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=\frac{2}{x+y}$.

c) Evaluate $\int_{-1}^{2} \int_{x^2}^{x+2} dy\, dx$.

d) Show that u is harmonic function if $u = \log(x^2 + y^2)$.

e) Change the order of integration $\int_{-1}^{1} \left[\int_{-\sqrt{1-x^2}}^{1-x^2} f\, dy \right] dx$.

f) Prove that if f has an extremum at a point (a, b) and if the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ exist in a neighborhood of (a, b) then $f_x(a,b) = 0 = f_y(a,b)$.

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