S.Y. B. SC. (Computer Science) SEM –III (CBCS - 2016 COURSE) : WINTER - 2018
SUBJECT : LINEAR ALGEBRA

Day : Tuesday
Date : 16/10/2018

W-2018-0914

Time : 11.00 AM TO 02.00 PM
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) Let V be a vector space, $\bar{u}$ be a vector in V and c be any scalar. Prove that:
i) $0.\bar{u} = \bar{0}$ **ii)** $c\bar{0} = \bar{0}$ **iii)** $(-1)\bar{u} = \overline{-u}$

b) Show that the vectors $\bar{u} = (0, 3, 1, -1)$, $\bar{v} = (6, 0, 5, 1)$ and $\bar{w} = (4, -7, 1, 3)$ form a linearly dependent set.

c) Determine whether or not set S is basis for V = $\mathbb{R}^3$ where, S = {(1, 1, 1), (2, 2, 0), (3, 0, 0).

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) Prove that if λ is an eigen value of a square matrix A, then λ^m is an eigen value of A^m for every positive integer m.

b) Find matrix P that diagonalizes the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 5 \\ 0 & 4 & 5 & 1 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix}$.

c) Find all the eigen values of A and find the eigen space corresponding to the smallest and largest eigen value of matrix,
$$A = \begin{bmatrix} 4 & 6 & 6 \\ 1 & 3 & 2 \\ -1 & -4 & -3 \end{bmatrix}.$$

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2 × 2 matrix, then show that if A is invertible, then $ad - bc \neq 0$ and in this case $A^{-1} = \dfrac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

b) For what value of 'a' does the following system has:
i) Unique solution or trivil solution.
ii) Infinitely many solutions.
$$(a-3)x + y = 0$$
$$x + (a-3)y = 0$$

P.T.O.

c) Solve the following system of equation by Gauss Jordan elimination method.

$$x_1 + 3x_2 + x_3 + x_4 = 3$$
$$2x_1 - 2x_2 + x_3 + 2x_4 = 8$$
$$3x_1 + x_2 + 2x_3 - x_4 = -1$$

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) If L : V $\to$ W is a linear transformation then prove that the range L is a subspace of W.

b) Let L : $\mathbb{R}^3 \to \mathbb{R}^2$ be a linear transformation such that L(1, 0, 0) = (0, 0), L(0, 1, 0) = (1, 1), L(0, 0, 1) = (1, -1) then compute L(4, -1, -1).

c) Find standard matrix for linear transformation T : $\mathbb{R}^3 \to \mathbb{R}^5$ defined as, T $(x, y, z) = (2x + y - z, -y + 2z, x - z, x + y - z, 2x)$.

d) Let L : $\mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by $L\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+c \\ a+b \\ b-c \end{bmatrix}$

then find a basis for ker L.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) If $\begin{bmatrix} x+y & z+w \\ z-w & z-y \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 10 & 2 \end{bmatrix}$ then find x, y, z, w.

b) Why the set {(1, 2), (2, 1), (4, 5)} is linearliy dependent subset of R^2?

c) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, also find eigen values of A^4.

d) Prove that $L(\bar{v}) = 0, \forall \bar{v} \in V$ is a linear transformation where V be a vector space.

e) Find the dot product of $\bar{u} = \begin{bmatrix} 2 \\ -3 \\ 5 \\ 4 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ 4 \end{bmatrix}$.

f) State whether the following statement is true or false. "Union of any two subspaces W_1 and W_2 of vector space V is also a subspace of V". justify.

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