S.Y.B.SC. (Computer Science) SEM –III (2014 COURSE) : WINTER - 2018

SUBJECT : LINEAR ALGEBRA

Day : Saturday
Date : 13/10/2018

W-2018-0955

Time 12.00 NOON TO 02.00 PM
Max. Marks : 40

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Reduce matrix A in row echelon form where

$$A = \begin{bmatrix} 0 & 4 & 5 & 3 \\ 2 & 1 & 0 & 2 \\ 1 & 2 & 5 & 6 \end{bmatrix}$$

b) Find LU factorization of the co-efficient matrix of the given linear system $A\bar{x} = \bar{b}$. Where

$$A = \begin{bmatrix} 2 & 3 & 0 & 1 \\ 4 & 5 & 3 & 3 \\ -2 & -6 & 7 & 7 \\ 8 & 9 & 5 & 21 \end{bmatrix}$$

c) Show that the set $S = \{t^2 + 1, t - 1, 2t + 2\}$ is a basis for the vector space P_2.

Q.2 Attempt **ANY TWO** of the following: **(10)**

a) Prove that if V is a vector space then

i) If $c\bar{u} = \bar{0}$ then $c = 0$ or $\bar{u} = \bar{0}$

ii) $(-1)\bar{u} = -\bar{u}$ for every $\bar{u}$ in V

b) Determine value of a for which system have

i) No solution

ii) unique solution

$$x_1 + ax_2 = 4$$

$$ax_1 + 9x_2 = 5$$

c) Let L: $P_2 \rightarrow P_2$ be a linear transformation defined by $L(at^2 + bt + c) = (a + c)t^2 + (b + c)t$

i) Is $2t^2 - t$ in Range of L?

ii) Find basis for Kernel of L

P.T.O

Q.3 Attempt **ANY TWO** of the following: **(10)**

a) Prove that if λ is an eigen value of a square matrix A, then λ^m is an eigen value of A^m for every positive integer m.

b) Find all eigen value of A and eigen space corresponding to the smallest eigen value of the matrix A where

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 2 & 3 & 2 \\ 3 & 3 & 4 \end{bmatrix}$$

c) Let $L : IR^2 \rightarrow IR^2$ be a linear transformation for which
$L(1,1) = (1, -2)$, $L(-1, 1) = (2, 3)$

i) What is $L(-1, 5)$?

ii) What is $L(a, b)$?

Q.4 Attempt **ANY FIVE** of the following: **(10)**

a) Find x if $\bar{a} = \begin{bmatrix} 1 \\ x \\ 3 \end{bmatrix}$ & $\bar{b} = [2 \quad 4 \quad 9]$ also $\bar{a} \cdot \bar{b} = 37$

b) Let $W = \{(x, y) \in IR^2 / x + y = 100\}$. Is W a subspace of IR^2 with usual addition and scalar multiplication? Justify.

c) Are the vectors $\overline{V_1} = (1,0,1,2), \overline{V_2} = (0,1,1,2), \& \overline{V_3} = (1,1,1,3)$ in IR^4 linearly dependent or independent?

d) Define the terms :

i) Basis of vector space **ii)** Dimension of vector space

e) Find the characteristics polynomial of the following matrix.

$$\begin{bmatrix} 10 & -9 \\ 4 & -2 \end{bmatrix}$$

f) Let $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ find the eigen values of A^4.

g) A mapping $T : R^2 \rightarrow R^3$ is defined as $T(x, y) = (x + y, x - y, 1)$. Determine whether T is linear transformation.

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