F.Y. B. SC. (Computer Science) SEM –II (CBCS - 2016 COURSE) :
WINTER - 2018
SUBJECT : ALGEBRA – II

Day : Monday
Date : 15/10/2018

W-2018-0906

Time : 03.00 PM TO 06.00 PM
Max. Marks : 60

N. B. :

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 A) Choose the correct alternative: **(06)**

i) Let G be a group and $a,b,c,\in G$, if $a\cdot b = a\cdot c$ then _____.
a) $a\cdot b = a$ **b)** $b = c$ **c)** $a = c$ **d)** $b = c$

ii) In Z_7, $(\overline{11}+\overline{3})$ = _____.
a) $\overline{0}$ **b)** $\overline{1}$ **c)** $\overline{7}$ **d)** $\overline{3}$

iii) Subgroup of an abelian group is _______.
a) semi group **b)** non abelian **c)** abelian **d)** none of these

iv) Sn is non abelian for ________.
a) $n \geq 3$ **b)** $n \leq 3$ **c)** $n = 0$ **d)** $n \geq 2$

v) A cycle of length two is called ___.
a) disjoint cycle **b)** transposition
c) permutation cycle **d)** none of these

vi) A group having no proper normal subgroup is called _______.
a) normal group **b)** quasi group **c)** simple group **d)** cyclic group

B) Answer all the following questions: **(06)**

i) Prove that if a is any element in a group G then $(a^{-1})^{-1} = a$.

ii) Show that $(Z_5, +_5)$ is a cyclic group.

iii) Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 3 & 7 & 6 \end{pmatrix}$, find order of σ .

iv) Define : homomorphism.

v) Define a normal subgroup.

vi) Define a field.

P.T.O.

Q.2 Attempt any **THREE** of the following questions: **(12)**

a) Show that $(Z_7^*, +_7)$ is a group.

b) Find the order of every element in $(Z_6, +_6)$.

c) Show that every cyclic group is abelian.

d) Find all the subgroups of a cyclic group $G = \{a, a^2, a^3, ..., a^{30} = e\}$.

Q.3 Attempt any **FOUR** of the following: **(12)**

a) Given : $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}, g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}, h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$

Compute : **i)** fog **ii)** (fog)oh

b) In the union of two subgroups of a group is subgroup again? Justify.

c) Show that $(Z, +)$ is isomorphic to $(m\mathbb{Z}, +)$.

d) Compute the indicated product of cycle that is permutation on $\{1,2,3,4,5,6\}$ where $\sigma = (1\ 3\ 5)\ (2\ 4\ 6)\ (1\ 2)$.

e) Prove that every subgroup of an abelian group is normal.

Q.4 Attempt any **TWO** of the following: **(12)**

a) Let G be a group, if $a, b \in G$ then prove that $(a \cdot b)^{-1} = b^{-1}a^{-1}$.

b) Let G be group then prove that :

i) Identify element in G is unique.

ii) Every element of G has unique inverse.

c) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$ as a product of disjoint cycles. Determine whether σ is even or odd. Find σ^{-1}.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) A subgroup H of a group G is normal if and only if $xHx^{-1} = H \ \forall x \in G$.

b) State and prove Lagrange's theorem.

c) Find all the subgroups of group Z_{36} and draw a lattice diagram for these subgroups.

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