F.Y. B. SC. (Computer Science) SEM – I (CBCS - 2016 COURSE) :
WINTER - 2018
SUBJECT : ALGEBRA – I

Day : Wednesday
Date : 17/10/2018

W-2018-0897

Time : 11.00 AM TO 02.00 PM
Max. Marks : 60

N. B. :

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 A) Select the correct alternative: **(06)**

i) If $R = \phi$, R is known as ______.

a) Universal Relation **b)** Identity Relation
c) Void Relation **d)** Reflexive Relation

ii) In $\mathbb{Z}_4, (\overline{10} + \bar{1}) =$ __________.

a) $\bar{3}$ **b)** $\bar{0}$ **c)** $\bar{1}$ **d)** $\bar{2}$

iii) The modulus of the complex number $z = x + iy$ is _________.

a) $x^2 + y^2$ **b)** $x + y$ **c)** $\sqrt{x^2 + y^2}$ **d)** 0

iv) A injective function means a function is ____.

a) Only one-one **b)** Only onto
c) Neither one-one nor onto **d)** Both one-one and onto

v) Least common multiple of (150, 35) = _______.

a) 5 **b)** 0 **c)** 1050 **d)** 35

vi) M(R) = the matrix of all 1's if and only if _____.

a) $R = A \times A$ **b)** $R = A - A$ **c)** $R = \frac{A}{B}$ **d)** $R = A + A$

B) Attempt **ALL** the following questions: **(06)**

i) Find the modulus of complex number $Z = \frac{1+i}{1-i}$.

ii) Define partial order relation.

iii) Prove that if $\gcd(b,c) = 1$, $a \mid b$ then $\gcd(a,c) = 1$.

iv) Check whether the function $f(x) = x^2$ is onto or not.

v) Define a parity check matrix.

vi) Find the Hamming distance between $x = 00000$ and $y = 11111$.

P.T.O.

Q.2 Attempt any **THREE** of the following: **(12)**

a) Prove that $2^{n+1} \leq 2^n$ for $n \geq 3$ by using first principle of mathematical induction.

b) Prove that $(a,m)=(b,m)=1$ then $(ab,m)=1$.

c) Prove that $|z_1+z_2| \leq |z_1|+|z_2| \ \forall z_1,z_2 \in \mathbb{C}$.

d) Construct a decoding table for the (2,4) codes given by the following generator matrix:

$$G=\begin{bmatrix}1 & 0 & 1 & 0\\ 0 & 1 & 0 & 1\end{bmatrix}$$

Q.3 Attempt any **FOUR** of the following: **(12)**

a) Obtain gof if $f:\mathbb{R} \to \mathbb{R}$ such that $f(x)=x^2-1$ and $g:\mathbb{R} \to \mathbb{R}$ such that $g(x)=3x+5$.

b) Let A = {a,b,c,d,e}. Let R be the relation on A whose M(R) is as follows:

$$M(R)=\begin{array}{c} \\ a\\ b\\ c\\ d\\ e\end{array}\begin{array}{c}\begin{array}{ccccc}a & b & c & d & e\end{array}\\ \begin{bmatrix}1 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 1 & 0\\ 0 & 0 & 0 & 1 & 1\\ 0 & 1 & 1 & 0 & 0\\ 1 & 0 & 0 & 0 & 0\end{bmatrix}\end{array}$$

Draw a digraph of R.

c) Solve $x^8-x^4+1=0$ by using De Moivre's theorem.

d) If $a|b$ and $a|c$ then $a|(bx+cy)$ for any integers x and y.

e) Show that $\dfrac{(\cos 3\theta+i\sin 3\theta)^5(\cos\theta-i\sin\theta)^3}{(\cos 5\theta+i\sin 5\theta)^7(\cos 2\theta-i\sin 2\theta)^5}=[\cos 13\theta-i\sin 13\theta]$

Q.4 Attempt any **TWO** of the following: **(12)**

a) Let R be a relation on $\mathbb{Z}$ define by $x\ R\ y$ if and only if $5x+6y$ is divisible by 11, for $x,y \in \mathbb{Z}$ then prove that R is equivalence relation.

b) Let A = {1, 2, 3} and let R = {(1,1), (1,2), (2,3), (1,3), (3, 1) (3,2)}. Compute the transitive closure of R denoted by R* using Warshall's algorithm.

c) Express $\cos^7\theta$ in terms of the cosines of multiple angles, using De-Moivre's theorem.

Q.5 Attempt any **TWO** of the following: **(12)**

a) If $a,b,x \in \mathbb{Z}, n \in$ N and $a \equiv b(\bmod n)$ then

i) $(a+x) \equiv (b+x)(\bmod n)$

ii) $ax \equiv bx(\bmod n)$

iii) $(a-x) \equiv (b-x)(\bmod n)$

b) Find the gcd of 1357 and 851 and express the gcd in the form 1357m +851n where $m,n \in \mathbb{Z}$.

c) Let $H=\begin{bmatrix}1 & 1 & 0 & 1 & 1 & 0 & 0\\ 1 & 0 & 1 & 1 & 0 & 1 & 0\\ 0 & 1 & 1 & 1 & 0 & 0 & 1\end{bmatrix}$ be a parity check matrix for Hamming (7,4) code. Decode the following received words 1100001, 1110111.

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