**B.TECH SEM - III (2007 COURSE) (COMPUTER ENG G.) /ELECTRICAL ENGG / ELECTRONIC ENGG./INF. TECH./BIOMEDICAL ENGG./ E & TC ENGG.) : SUMMER - 2018**

**SUBJECT: ENGINEERING MATHEMATICS – III**

Day: **Monday**
Date: **21/05/2018**

**S-2018-2567**

Time: **02.30 PM TO 05.30 PM**
Max. Marks: 80

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**N.B.:**

1) **Q. No. 1** and **Q. No. 5** are **COMPULSORY**. Out of the remaining attempt **ANY TWO** questions from Section – **I** and Section – **II**.
2) Figures to the right indicate **FULL** marks.
3) Answers to both the sections should be written in **SEPARATE** answer books.
4) Use of non programmable **calculator** is **ALLOWED**.
5) Draw neat and labeled diagrams **WHEREVER** necessary.
6) Assume suitable data, if necessary.

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**SECTION - I**

**Q.1**

**a)** Solve : $x^2 \dfrac{d^2y}{d^2x} - 4x\dfrac{dy}{dx} + 6y = x^5$. **(05)**

**b)** Find the Fourier sine transform of **(04)**

$$f(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & x > 1 \end{cases}$$

**c)** If $v = 3x^2y - y^3$, find its harmonic conjugate u. Find f(z) = u + iv in terms of z. **(05)**

**Q.2** Solve **Any Three:** **(13)**

*i)* $(D^2 - 4D + 3)y = x^2 e^{2x}$.

*ii)* $(D^2 - D + 1)y = x^3 - 3x^2 + 1$

*iii)* $(D^3 + D)y = \cos x$

*iv)* $(D^2 + 1)\, y = \sec x$ (by the method of variation of parameter.)

**Q.3**

**a)** Evaluate $\oint_C \log z\,dz$, where 'C' is the circle |z| = 1. **(04)**

**b)** Evaluate $\displaystyle\int_0^{2\pi} \frac{\sin 2\theta}{5 + 4\cos\theta}\, d\theta$ **(05)**

**c)** Find the bilinear transformation, which maps the points $z = -1, 0, 1$ on to the points $w = 0,\ i,\ 3i$ . **(04)**

**Q.4**

**a)** Find the Fourier cosine transform of the function $f(x) = e^{-mx}\ (m > 0,\ x > 0)$ **(05)**

And hence prove that $\displaystyle\int_0^{\infty} \frac{\cos\lambda x}{\lambda^2 + m^2}\, d\lambda = \frac{\pi}{2m} e^{-mx}$ .

**P.T.O.**

b) Using inverse Fourier cosine transform, find $f(x)$, if **(04)**

$$F_c(\lambda) = \begin{cases} \sqrt{\frac{2}{\pi}}\left(a - \frac{\lambda}{2}\right), & \lambda \le 2a \\ 0, & \lambda > 2a \end{cases}$$

c) Find $Z\{f(k)\}$ where $f(k) = \sin\left(\frac{k\pi}{4} + \alpha\right),\ k \ge 0$ **(04)**

## SECTION - II

**Q.5** a) Find the Laplace transform of the following: **(06)**

$i)\ \frac{d}{dt}\left(\frac{\sin t}{t}\right)$ $\qquad ii)\ \cos^2 t.$

b) A vector field is given by $\vec{F} = \cos y\ \vec{i} + x(1 - \sin y)\vec{j}$, evaluate. **(04)**

$\int_C \vec{F}.\vec{dr}$ where C is the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, z = 0.

c) If the directional derivative of $\phi = axy + byz + czx$ at $(1, 1, 1)$ has maximum magnitude 8 in a direction parallel to Y- axis, find the values of a, b, c. **(04)**

**Q.6** a) Find inverse Laplace transform of $\frac{2s + 5}{s^2 + 4s + 13}$ **(04)**

b) Use the convolution theorem to find Inverse Laplace transform of **(04)**

$\frac{1}{s(s^2 + a^2)}$

c) Solve the differential equation $\frac{d^2y}{dt^2} + y = t,\ \ y(0) = 1,\ \ y'(0) = -2$ **(05)**

**Q.7** a) Show that $\vec{F} = \left(ye^{xy}\cos z\right)\vec{i} + \left(xe^{xy}\cos z\right)\vec{j} - \left(e^{xy}\sin z\right)\vec{k}$ *is* irrotational. **(05)**

Find $\phi$ *if* $\overline{F} = \nabla\phi$.

b) Prove that $\nabla.\frac{\overline{r}}{r^3} = 0$ **(04)**

c) Prove that $\vec{F} = \frac{1}{x^2 + y^2}\left(x\,\vec{i} + y\,\vec{j}\right)$ is solenoidal. **(04)**

**Q.8** a) Evaluate $\iint_S \left(x^2y^3\,\vec{i} + z^2x^3\,\vec{j} + x^2y^3\,\vec{k}\right).\vec{ds}$ **(06)**

Where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

b) Verify stoke's theorem for $\vec{F} = -y^3\,\vec{i} + x^3\,\vec{j}$ and the closed C is the boundary of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$ **(07)**

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