B.TECH SEM – IV (2007 COURSE) (MECHANICAL ENGG.)/ (PRODUCATION ENGG.) : SUMMER - 2018

SUBJECT: ENGINEERING MATHEMATICS-III

Day : **Saturday** | Date : **02/06/2018** | **S-2018-2630** | Time : **10.00 AM TO 01.00 PM** | Max. Marks: 80

N. B. :

1) **Q. No.1** and **Q. No.5** are **COMPULSORY.** Out of the remaining attempt **ANY TWO** questions from each Section.
2) Figures to the right indicate **FULL** marks.
3) Answers to both the sections should be written in **SEPARATE** answer book.
4) Use of non-programmable **CALCULATOR** is allowed.
5) Assume suitable data, if necessary.

SECTION-I

Q.1 a) Solve: $\frac{dx}{y^2 z}=\frac{dy}{x^2 z}=\frac{dz}{y^2 x}$. **(04)**

b) An electric circuit consists of an inductance L, a condenser of capacitance C and an emf $E = E_0 \cos\omega t$, so that the charge Q satisfies differential equation **(05)**

$$\frac{d^2Q}{dt^2}+\frac{Q}{CL}=\frac{E_0}{L}\cos\omega t$$

If $\omega=\frac{1}{\sqrt{CL}}$ and initially at t = 0, $Q = Q_0$ and the current $i = i_0$, show that the charge Q at time t is given by $Q=Q_0\cos\omega t+\frac{i_0}{\omega}\sin\omega t+\frac{E_0}{2L\omega}t\sin\omega t$.

c) Find Laplace transform of: **(05)**

$$f(t)=\begin{cases} t, & 0<t<\pi \\ \pi-t, & \pi<t<2\pi \end{cases} \quad \text{and } f(t+2\pi)=f(t).$$

Q.2 Solve **ANY THREE** of the following: **(13)**

a) $(D^2-D-6)y=e^x\cosh 2x$.

b) $(D^3+D^2-D-1)y=\cos 2x$.

c) $(D^2+3D+2)y=\sin(e^x)$.

d) $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x\log x$.

e) $(D^2-1)y=\frac{2}{1+e^x}$ (by variation of parameters)

Q.3 a) A string is stretched and fastened to two points 'l' apart. Initial displacement $u(x,0)=a\sin\frac{\pi x}{l}$ it is released at time t = 0. Find the displacement u(x, t). satisfies $\frac{\partial^2 u}{\partial t^2}=a^2\frac{\partial^2 u}{\partial x^2}$. **(06)**

b) System of differential equations of an undamped mechanical system is given by. **(07)**

$\ddot{x}_1=-x_1-3(x_1-x_2)$

$3\ddot{x}_2=3(x_1-x_2)-3x_2$

Assuming that there is no friction , find the natural frequencies of the system and corresponding normal modes of vibration using matrix method.

P.T.O.

Q.4 **a)** Obtain Fourier sine and cosine transform of the function $f(x)=x^{m-1}$. **(07)**

b) Find inverse Laplace transform of: **(06)**

i) $\frac{1}{s}\log\left(\frac{s^2+9}{s^2+16}\right)$ **ii)** $\frac{s+7}{s^2+2s+2}$

SECTION-II

Q.5 **a)** A can hit the target 1 out of 4 times, B can hit the target 2 out of 3 times, C can hit the target 3 out of 4 times. Find the probability of at least two of them hit the target. **(04)**

b) Show that $\overline{F}=f(r)\overline{r}$ is always irrotational and find f(r) such that it is solenoidal. **(05)**

c) Evaluate $\int_C \overline{F}\cdot d\overline{r}$, $\overline{F}=x^2\hat{i}+xy\hat{j}$ where C: y = x^2 and the line y = x. **(05)**

Q.6 **a)** Find the correlation coefficient between x and y for the given values. Find also the two regression lines. **(08)**

x	1	2	3	4	5	6	7	8	9	10
y	10	12	16	28	25	36	41	49	40	50

b) Assuming that the diameter of 100 brass plugs taken consecutively from a machine form a normal distribution with mean 0.7515cm and standard deviation 0.0020cm. How many of the plugs are likely to rejected, if the acceptable diameter is 0.752 ± 0.004 cm? **(05)**
(Given: z_1 = 2.25, A_1 = 0.4878, z_2 = 1.75, A_2 = 0.4599)

Q.7 **a)** Show that $\overline{F}=yz\hat{\imath}+xz\hat{j}+xy\hat{k}$ is irrotational and find a scalar function ϕ such that $\overline{F}=\nabla\phi$. **(05)**

b) Prove that: **(08)**

i) $\nabla^2\left[\nabla.\left(\frac{\overline{r}}{r^2}\right)\right]=\frac{2}{r^4}$.

ii) $\nabla\left(\frac{\overline{a}.\overline{r}}{r^n}\right)=\frac{\overline{a}}{r^n}-\frac{n(\overline{a}.\overline{r})}{r^{n+2}}\overline{r}$.

Q.8 **a)** If $\overline{F}=(2x^2-3z)\hat{i}-2xy\hat{j}-4x\hat{k}$, then evaluate $\iiint_V\left(\nabla.\overline{F}\right)dV$, where V is solid bounded by the planes x = 0, y = 0, z =0 and 2x+2y+z = 4. **(07)**

b) A vector field is given by: $\overline{F}=\sin y\,\hat{\imath}+x(1+\cos y)\,\hat{\jmath}$ **(06)**

Evaluate: $\int_c \overline{F}.d\overline{r}$, where C is $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

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