

B.TECH SEM - IV (2007 COURSE) (CHEMICAL ENGG.) :
SUMMER - 2018
SUBJECT : ENGINEERING MATHEMATICS - III

Day : **Saturday**
 Date : **02/06/2018**

S-2018-2600

Time : **10.00 AM TO 01.00 PM**
 Max. Marks : 80

N. B. :

- 1) **Q. No. 1 and Q. No. 5 are COMPULSORY.** Out of remaining attempt **ANY TWO** questions from each section.
- 2) Figures to the right indicate **FULL** marks.
- 3) Answers to both the sections should be written in the **SEPARATE** answer books.
- 4) Use of non-programmable **CALCULATOR** is allowed.
- 5) Assume suitable data, if necessary.

SECTION - I

Q. 1 a) Solve by method of variation of parameters: **(05)**

$$(D^2 - 1)y = \frac{2}{1 + e^x}$$

b) Solve: **(05)**

$$\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{y^2 x}$$

c) Express the $f(x) = \begin{cases} 1, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$ as Fourier integral. **(04)**

Q. 2 Solve **ANY THREE** of the following: **(13)**

- a) $(D^3 + 1)y = \cos(2x - 1)$
- b) $(D^2 - 2D + 1)y = xe^x \sin x$
- c) $(D^4 + 2D^2 + 1)y = x^2 \cos x$
- d) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} + y = \log x$
- e) $(1 + x)^2 \frac{d^2 y}{dx^2} + (1 + x) \frac{dy}{dx} + y = 2 \sin(\log(1 + x))$

Q. 3 a) The differential equation for the displacement y of a whirling shaft when the weight of the shaft is taken into account is **(07)**

$$EI \frac{d^4 y}{dx^4} - \frac{W\omega^2}{g} y = W$$

Taking shaft of length $2l$ with the origin at the centre and short bearings of both ends, show that the maximum deflection of the shaft is

$$\frac{g}{2\omega^2} (\sec hal + \sec al - 2)$$

b) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions **(06)**

$$u(0, y) = u(l, y) = u(x, 0) = 0 \text{ and } u(x, a) = \sin \frac{n\pi x}{l}.$$

P. T. O.

Q. 4 a) Find the Fourier sine and cosine transform of x^{n-1} , $n > 0$. (07)

b) Solve the integral equation $\int_0^\infty f(x) \sin \lambda x \, dx = \begin{cases} 1, & 0 \leq \lambda \leq 1 \\ 2, & 1 \leq \lambda \leq 2 \\ 0, & \lambda \geq 2 \end{cases}$ (06)

SECTION - II

Q. 5 a) Find the Laplace transform of: (05)

i) $t^2 \sin at$

ii) $\sin \sqrt{t}$

b) Find the tangential and normal components of acceleration at any time t for the curve $\vec{r} = at \cos t \hat{i} + at \sin t \hat{j}$ (05)

c) Evaluate $\oint (\cos y \hat{i} + x(1 - \sin y) \hat{j}) \cdot d\vec{r}$ for a closed curve which is given by $x^2 + y^2 = 1$, $z = 0$. (04)

Q. 6 a) Find Laplace transform of: $\int_0^t \int_0^t \int_0^t (t \sin t) \, dt \, dt \, dt$ (05)

b) Find the inverse Laplace transform of: $\frac{s+2}{s^2(s+1)(s+2)}$ (04)

c) Solve by Laplace transform: $\frac{d^2x}{dt^2} + 9x = \cos 2t$, if $x(0) = 1$, $x(\pi/2) = -1$. (04)

Q. 7 a) Find the directional derivative of $\phi = e^{2x} \cos yz$ at $(0, 0, 0)$ in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \frac{\pi}{4}$ (05)

b) Prove that: (08)

i) $\nabla \times \left[\frac{\vec{a} \times \vec{r}}{r^3} \right] = -\frac{\vec{a}}{r^3} + \frac{3(\vec{a} \cdot \vec{r})}{r^5} \vec{r}$

ii) $\nabla \cdot \left[r \nabla \left(\frac{1}{r^n} \right) \right] = \frac{n(n-2)}{r^{n+1}}$

Q. 8 a) Evaluate: (07)

$\iint_S 2x^2 y \, dy \, dz - y^2 \, dz \, dx + 4xz^2 \, dx \, dy$ over the curved surface of the cylinder $y^2 + z^2 = 9$, bounded by $x = 0$ and $x = 2$.

b) Verify Stokes theorem for $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ over the surface of a cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $z = 2$ above the xy plane (open at the bottom) (06)