B.TECH. SEM -I (CHEMICAL/ CIVIL/ ELECTRICAL/ MECHANICAL/ PRODUCTION/ COMPUTER/ INFO. TECH./ ELECTRONICS / BIO MEDICAL / E & TC) 2014 COURSE (CBCS) : SUMMER - 2018

SUBJECT : ENGINEERING MATHEMATICS – I

Day : **Monday**	**S-2018-2204**	Time : **10.00 AM TO 01.00 PM**
Date : **21/05/2018**		Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Assume suitable data if necessary.

Q.1 a) Investigate the values of λ *and* μ so that the system **[10]**

$$x+2y+\lambda z=1$$
$$x+2\lambda y+z=\mu$$
$$\lambda x+2y+z=1$$

has **i)** no solution **ii)** only one solution **iii)** Infinitely many solutions

OR

b) Find Eigen values and Eigen vectors for the following matrix. **[10]**

$$A=\begin{bmatrix}1 & -1 & 0\\ 1 & 2 & 1\\ -2 & 1 & -1\end{bmatrix}$$

Q.2 a) If α and β are the roots of $x^2-2x+2=0$ **[10]**

then prove that $\alpha^n+\beta^n=2^{\frac{n}{2}+1}\cos\left(\frac{n\pi}{4}\right)$

OR

b) If tan (A + iB) = x + iy then prove that **[10]**

i) $x^2+y^2+2x\cot 2A=1$

ii) $x^2+y^2-2y\coth 2B+1=0$

Q.3 If $y=a\cos(\log x)+b\sin(\log x)$ then prove that **[10]**

$x^2y_{n+2}+(2n+1)xy_{n+1}+(n^2+1)y_n=0$.

OR

Prove that $\cos^{-1}(\tanh(\log x))=\pi-2\left(x-\frac{x^3}{3}+\frac{x^5}{5}+......\right)$ **[10]**

P.T.O.

Q.4 a) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{(\log n)^n}$ **[05]**

b) Evaluate : $\lim_{\theta \to \frac{\pi}{2}} \frac{\log\left(\theta - \frac{\pi}{2}\right)}{\tan\theta}$. **[05]**

OR

a) Test the convergence of the series $\sqrt{\frac{1}{2^3}} + \sqrt{\frac{2}{3^3}} + \sqrt{\frac{3}{4^3}} + \ldots\ldots.$. **[05]**

b) Evaluate : $\lim_{x \to \frac{1}{2}} \frac{\cos^2 \pi x}{e^{2x} - 2xe}$. **[05]**

Q.5 If $z = x^n f\left(\frac{y}{x}\right) + x^{-n}\phi\left(\frac{x}{y}\right)$, then prove that **[10]**

$x^2 z_{xx} + 2xy\, z_{xy} + y^2 z_{yy} + xz_x + yz_y = n^2 z.$

OR

If $x = \sqrt{uv},\ y = \sqrt{uw},\ z = \sqrt{uv}$ and ϕ is a function of x, y, z then prove that **[10]**

$x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y} + z\frac{\partial\phi}{\partial z} = u\frac{\partial\phi}{\partial u} + v\frac{\partial\phi}{\partial v} + w\frac{\partial\phi}{\partial w}$

Q.6 If $x = \sqrt{vw},\ y = \sqrt{uw},\ z = \sqrt{uv}$ and **[10]**

$u = r\sin\theta\cos\phi,\ v = r\sin\theta\sin\phi,\ w = r\cos\theta$ then find $\frac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$.

OR

Find the maximum and minimum values of **[10]**

$f(x,y) = x^3 + 3xy^2 - 3x^2 - 3y^2 + 4.$

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