B.TECH. SEM -I (2007 COURSE) (ALL BRANCHES) :
SUMMER - 2018
SUBJECT: ENGINEERING MATHEMATICS – I

Day: **Monday**
Date: **21/05/2018**

S-2018-2547

Time: **10.00 AM TO 01.00 PM**
Max Marks : 80

N.B. :

1) **Q.No.1** and **Q.No.5** are **COMPULSORY**. Out of the remaining questions attempt **ANY TWO** questions from each section.
2) Answers to both the sections should be written in the **SEPARATE** answer books.
3) Use of non-programmable **CALCULATOR** is allowed.
4) Figures to the right indicate **FULL** marks.
5) Assume suitable data if necessary.

SECTION – I

Q.1 **a)** Find rank of matrix A, where $A=\begin{bmatrix}2 & -2 & 0 & 6\\ 4 & 2 & 0 & 2\\ 1 & -1 & 0 & 3\\ 1 & -2 & 1 & 2\end{bmatrix}$ **[05]**

b) Find the modulus and amplitude of: $1-\cos\alpha+i\sin\alpha$. **[05]**

c) Find nth derivative of: $\dfrac{x}{(x+1)^4}$. **[04]**

Q.2 **a)** Examine for consistency and if consistent, then solve it: **[05]**

$$2x_1+x_2+5x_3+x_4=5$$
$$x_1+x_2-3x_3-4x_4=-1$$
$$3x_1+6x_2-2x_3+x_4=8$$
$$2x_1+2x_2+2x_3-3x_4=2$$

b) Examine for linear dependence or independence the following system of vectors. If dependent, find the relation between them. **[04]**
$x_1=(2,\,3,\,4,\,-2)$, $x_2=(-1,\,-2,\,-2,\,1)$, $x_3=(1,\,1,\,2,\,-1)$.

c) Show that $A=\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ is orthogonal matrix. **[04]**

Q.3 **a)** If $\dfrac{z-1}{z+i}$ is purely imaginary, find locus of z. **[04]**

b) A square lies entirely in second quadrant. If one of the side join the points -2 and 2i, find the complex numbers representing other vertices. **[05]**

c) If $\cos(\alpha+i\beta)\cos(\gamma+i\delta)=1$, prove that $\tanh^2\beta\cosh^2\delta=\sin^2\gamma$ **[04]**

Q.4 **a)** If $y=e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. **[05]**

b) If in the Cauchy's mean value theorem, $f(x)=e^x$ and $g(x)=e^{-x}$, show that c is the arithmetic mean between a and b on [a, b]. **[04]**

c) Find the nth derivative of : $\log(x^2+a^2)$. **[04]**

P.T.O.

SECTION – II

Q.5 **a)** Expand $\sqrt{1+\sin x}$ upto x^6. **[05]**

b) If $u=\sin\left(\sqrt{x}+\sqrt{y}\right)$, prove that **[04]**

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\left(\sqrt{x}+\sqrt{y}\right)\cos\left(\sqrt{x}+\sqrt{y}\right).$$

c) Examine for the functional dependence $u=\sin^{-1}x+\sin^{-1}y,\ v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ and find the relation between them, if exists. **[05]**

Q.6 **a)** Test the series for convergence whose n[th] term is given by $u_n=\sqrt{n^3+1}-\sqrt{n^3}$. **[05]**

b) Test for convergence the series: $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\ldots\ldots$ **[04]**

c) Using Taylors theorem, express $(x-2)^4-3(x-2)^3+4(x-2)^2+5$ in powers of x. **[04]**

Q.7 **a)** Find a, b if $\lim\limits_{x\to 0}\frac{a\sin^2 x+b\log\cos x}{x^4}=-\frac{1}{2}$. **[05]**

b) If $V=e^{\frac{r-x}{l}}$ where $r^2=x^2+y^2, l$ is a constant; show that $V_{xx}+V_{yy}+\frac{2}{l}V_x=\frac{V}{lr}$. **[04]**

c) If $(\cos x)^y=(\sin y)^x$ then find $\frac{dy}{dx}$. **[04]**

Q.8 **a)** If $x=e^v\sec u,\ y=e^v\tan u$, verify $JJ'=1$. **[04]**

b) Find the percentage error in the area of an ellipse when an error of 1% is made in measuring its major and minor axes. **[04]**

c) Examine for minimum and maximum values of : $\sin x+\sin y+\sin(x+y)$. **[05]**

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