S.Y.B.SC. SEM – IV (2014 COURSE) : SUMMER - 2018
SUBJECT : MATHEMATICS : VECTOR CALCULUS (M-41)

Day : **Tuesday** Time : **03.00 PM TO 05.00 PM**
Date : **24/04/2018** **S-2018-0727** Max. Marks : 40

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Prove that a non-constant vector function $\vec{u}(t)$ on $[a,b]$ is of constant direction if and only if $\vec{u}\times\frac{d\vec{u}}{dt}=\vec{0}$.

b) If $\vec{u}=x^2yz\hat{i}-2xz^3\hat{j}+xz^2\hat{k}$ and $\vec{v}=2z\hat{i}+y\hat{j}-x^2\hat{k}$, find $\frac{\partial^2(\vec{u}\times\vec{v})}{\partial x\partial y}$ at (1,0,-2).

c) Find $\phi(x,y,z)$ if $\nabla\phi=\left(y^2-2xyz^3\right)\hat{i}+\left(3+2xy-x^2z^3\right)\hat{j}$ $+\left(6z^3-3x^2yz^2\right)\hat{k}$ and $\phi(1,1,2)=0$

Q.2 Attempt any **TWO** of the following: **(10)**

a) If $\overline{u}$ is a vector point function and ϕ is a scalar point function then show that $\nabla\times\left(\phi\overline{u}\right)=(\nabla\phi)\times\overline{u}+\phi\left(\nabla\times\overline{u}\right)$.

b) Find the directional derivative of $\phi=x^2y^3-2xz^2+3$ at the point $P((2,1,-2)$ in the direction towards $Q(4,0,3)$

c) Evaluate $\iint_S \overline{f}\cdot\overline{n}\,dS$, where $\overline{f}=\left(x+y^2\right)\hat{i}-2x\hat{j}+2yz\hat{k}$ and S is the part of the plane $2x+y+2z=6$ in the first octant.

Q.3 Attempt any **TWO** of the following: **(10)**

a) Prove that a vector field $\overline{f}$ is conservative if and only if the circulation of $\overline{f}$ about any closed curve in the region is zero.

b) Evaluate by using Green's theorem $\oint_C\left[(2xy-x^2)\,dx+(x+y^2)dy\right]$, where C is the closed curve of the region bounded by $y=x^2$ and $y^2=x$.

c) Evaluate $\iint_S(x^3dy\,dz+x^2y\,dz\,dx+x^2z\,dx\,dy)$ by using divergence theorem, where S is the closed surface bounded by the planes $z=0$, $z=b$ and the circle $x^2+y^2=a^2$.

P.T.O.

Q.4 Attempt any **FIVE** of the following: **(10)**

a) If $\vec{v}(t)$ is differentiable at $t = t_0$ then show that it is continuous at $t = t_0$.

b) If $\bar{r} = \cos xy\,\hat{i} + (3xy - 2x^2)\hat{j} - (3x + 2y)\hat{k}$, find $\frac{\partial^2 \bar{r}}{\partial x \partial y}$.

c) Prove that $\nabla r^n = nr^{n-2}\bar{r}$ where $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

d) Prove that div $(\text{curl}\,\bar{u}) = 0$

e) The velocity $\bar{v}$ of a particle at any time $t \geq 0$ is given by $\bar{v} = \left(1 - e^{-t}\right)\hat{i} - \left(3t^2 + 6t\right)\hat{j} + (3 - 3\cos t)\hat{k}$. Find the displacement $\bar{r}$ at $t = 0$.

f) Define Curl of vector point function.

g) State Stoke's theorem.

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