

S.Y.B.SC. SEM – III (2014 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS: GROUP THEORY & DIFFERENTIAL EQUATIONS (M – 32)

Day : **Friday**
Date : **27/04/2018**

S-2018-0715

Time : **12.00 NOON TO 02.00 PM**
Max. Marks : 40

N.B.:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

- a) Let G be a group. If $a, b \in G$, then prove that $(b^{-1}ab)^m = b^{-1}a^mb, \forall m \in \mathbb{N}$.
- b) Show that the set $S = \{1, \omega, \omega^2\}$, where ω is a complex cube root of unity, forms a group under multiplication of complex numbers.
- c) Solve : $y = (1+p)x + p^2$, where $p = \frac{dy}{dx}$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

- a) Show that a non-empty subset H of a group G is a subgroup of G iff $ab^{-1} \in H, \forall a, b \in H$.
- b) Let G be the group of all real 2×2 matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ with $ad - bc \neq 0$; under matrix multiplication. Let $H = \left\{ \begin{bmatrix} a & b \\ 0 & d \end{bmatrix} \in G \mid ad \neq 0 \right\}$, then show that H is a subgroup of G .
- c) Solve the differential equation $p^3 - 2xy + 4y^2 = 0$, where $p = \frac{dy}{dx}$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

- a) Show that if $f(D)y = e^x$, then P.I. is $\frac{1}{f(D)} e^{ax} = \frac{e^{ax}}{f(a)}$, if $f(a) \neq 0$.
- b) Solve: $(D^3 + 1)y = e^{2x} \cdot \sin x$.
- c) Solve : $(D^2 - 4D + 3)y = \cos 3x + e^{2x}$.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

- a) Solve : $p^2 - p - 12 = 0$, where $p = \frac{dy}{dx}$.
- b) Define Clairaut's equation and explain the method of its solution.
- c) Solve : $\frac{d^2y}{dx^2} + 4y = 0$.
- d) Find particular integral of $(D^3 - 5D^2 + 8D - 4)y = e^{2x}$.
- e) Define order of an element in a group.
- f) Show that in a group identity element is unique.
- g) Find all the subgroups of a cyclic group of order 18.

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