**S.Y.B.SC. SEM – IV (2014 COURSE) : SUMMER - 2018**

**SUBJECT : MATHEMATICS : COMPLEX VARIABLES (M – 42)**

Day : **Friday** Time : **03.00 PM TO 05.00 PM**

Date : **27/04/2018** **S-2018-0729** Max. Marks : 40

---

**N.B.:**

**1)** All questions are **COMPULSORY**.

**2)** Figures to the right indicate **FULL** marks.

---

**Q.1** Attempt **ANY TWO** of the following: **[10]**

**a)** Show that if f(z) = u + iv is an analytic function of z, then f(z) is independent of z.

**b)** Find an analytic function, whose imaginary part is $4xy - x^3 + 3xy^2$.

**c)** Evaluate : $\lim\limits_{z \to e^{i\pi/3}} \dfrac{\left(z - e^{i\pi/3}\right) z}{z^3 + 1}$.

**Q.2** Attempt **ANY TWO** of the following: **[10]**

**a)** Using Cauchy's theorem obtain the value of $\int_C e^z \, dz$, where c is the circle $|z| = 1$ and deduce that $\int_0^{2\pi} e^{\cos\theta} . \cos(\theta + \sin\theta) \, d\theta = 0$.

**b)** Using Cauchy's integral formula evaluate $\int_C \dfrac{\sin^6 z \, dz}{(z - \pi/6)^3}$, where C is a circle $|z| = 1$.

**c)** Obtain the expansion of $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$, in the region $2 < |z| < 3$ .

**Q.3** Attempt **ANY TWO** of the following: **[10]**

**a)** State and prove Cauchy's residue theorem.

**b)** Evaluate by contour integration $\int_0^{2\pi} \dfrac{d\theta}{17 - 8\cos\theta}$.

**c)** Using contour integration evaluate $\int_0^{\infty} \dfrac{x^2 dx}{(x^2+1)(x^2+4)}$.

**Q.4** Attempt **ANY FIVE** of the following: **[10]**

**a)** Evaluate $\lim\limits_{z \to 1+i} \dfrac{z^4 + 4}{z^2 - 2i}$.

**b)** Obtain the Maclaurin's expansion of coshz.

**c)** Find the residue of $f(z) = \dfrac{z+3}{z^2(z^2+4)}$ at z = 2i.

**d)** Determine the poles and their orders for the function $f(z) = \dfrac{z^2 + 1}{(z^2+3)^2 (z^2-4)^3}$.

**e)** Evaluate $\int_C \dfrac{dz}{z}$, where C is the circle with centre at the origin and radius r.

**f)** Define Laurent's series.

**g)** State cauchy's theorem for an analytical function.

* * * *