S.Y.B.SC. SEM – IV (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT: MATHEMATICS: COMPLEX VARIABLES

Day : **Friday** Time: **11.00 A.M. TO 02.00 PM**
Date : **27/04/2018** **S-2018-0672** Max. Marks: 60.

N.B.:
1) All questions are **COMPULSORY.**
2) Figures to the **RIGHT** indicate full marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Show that a necessary condition that a function w = f(z) = u(x, y) + i v(x, y) be analytic at a point z = x + iy of its domain D is that at (x, y), the first order partial derivatives of u and v w.r.t. x and y exist and satisfy the Cauchy-Riemann equations $u_x = v_y$ and $u_y = -v_x$.

b) Find the analytic function whose real part is, $u = y^3 - 3x^2y$.

c) Evaluate : $\int_C \frac{z^3}{z-2i}dz$, where C is the circle |z - 2| = 5, by using Cauchy's integral formula.

Q.2 Attempt any **TWO** of the following: **(12)**

a) Using Cauchy's theorem obtain the value of $\int_C e^z dz$, where C is the circle |z| = 1 and deduce that $\int_0^{2\pi} e^{\cos\theta}\left[\sin(\theta+\sin\theta)\right]d\theta = 0$.

b) Find Laurent's series for

$$f(z) = \frac{3z-3}{(2z-3)(z-2)}$$

valid for $\frac{1}{2} < |z-1| < 1$

c) Evaluate by contour integration

$$\int_C \frac{3z^2+2}{(z-1)(z^2+9)}dz$$

where C is circle |z|=4.

Q.3 Attempt any **TWO** of the following: **(12)**

a) State and prove Cauchy's residue theorem.

b) Evaluate by contour integration $\int_0^{2\pi} \frac{d\theta}{5+3\cos\theta}$.

c) Prove that analytic function with constant argument is constant.

Q.4 Attempt any **THREE** of the following: **(12)**

a) Evaluate: $\lim_{z\to e^{i\pi/3}} \frac{\left(z-e^{\frac{i\pi}{3}}\right)z}{z^3+1}$.

b) Evaluate $\int_C (x^2+iy^3)dz$, where C is the line segment from z = 1 to z = i .

c) Verify Cauchy – Goursat theorem for f(z) = z + 2 taken around the unit circle | z | = 1.

d) Find residue of $f(z) = \frac{1}{z^2(z-i)}$ at z = i by expanding f(z) as a Laurent's series about z = i.

P.T.O.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Find the zeros of $(z^4 + 8z^2 + 16)(z^2 + z + 1)$.

b) Find $\int_C \frac{e^z}{(z+1)^2} dz$, where C is the circle $|z - 1| = 3$.

c) Show that the function $f(z) = \frac{z^2+1}{z^2-3z+2}$ is continuous for all points outside $|z| = 2$.

d) Prove that $\lim_{z \to 0} \frac{\bar{z}}{z}$ does not exists.

e) Define pole of the function with an example.

f) Find the residue of $f(z) = \frac{z^2}{(z-1)(z-2)(z-3)}$, at the simple pole.

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