F.Y.B.SC. SEM – I (2014 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS : CALCULUS (M-12)

Day : **Saturday**
Date : **28/04/2018**

S-2018-0689

Time : **12.00 NOON TO 02.00 PM**
Max. Marks : 40

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Prove that every continuous functions on closed and bounded interval attains its bounds.

b) Discuss the continuity of the following functions in the interval [0,8], where

$$f(x)=\frac{x^2+1}{x-2} \quad , \text{for } 0\le x\le 3$$
$$=3x-1 \quad , \text{for } 3<x\le 5$$
$$=\frac{x^2+x+1}{x-1} \quad , \text{for } 5<x\le 8$$

c) If $y=(x^2-1)^n$, then prove that $(x^2-1)y_{n+2}+2xy_{n+1}-n(n+1)y_n=0$.

Q.2 Attempt any **TWO** of the following: **(10)**

a) State and prove Cuachy's mean value theorem.

b) Verify Rolle's mean value theorem for the function $f(x)=x-x^3$, over $[-1,1]$.

c) Using Lagrange's mean value theorem, prove that $\frac{b-a}{1+b^2}<\tan^{-1}b-\tan^{-1}a<\frac{b-a}{1+a^2}$. Hence show that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\left(\frac{4}{3}\right)<\frac{\pi}{4}+\frac{1}{6}$

Q.3 Attempt any **TWO** of the following: **(10)**

a) Show that the following sequence is monotonic and bounded and also find its limit 0.5,0.55,0.555,0.5555,.....

b) Show that $\sum_{n=1}^{\infty}\frac{1}{n^p}$ is convergent if $p>1$.

c) Using Taylors theorem, prove that $\log\sec x=\frac{x^2}{2!}+\frac{2x^4}{4!}+\frac{16x^6}{6!}+....$

Q.4 Attempt any **FIVE** of the following: **(10)**

a) Show that function f defined by $f(x)=|x|$ is continuous but not differentiable.

b) Explain geometrical meaning of Lagrange's mean value theorem.

c) Evaluate $\lim_{x\to 0}\frac{xe^x-\log(1+x)}{x^2}$

d) If $y=\cos^3 x$, find y_n

e) If $y=\log(4x+3)+3^{7x}+e^{5x}$, find y_n

f) Discuss the convergence of $\sum_{n=1}^{\infty}\frac{n!}{2^n}$

g) Define removable and irremovable discontinuity.

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