S.Y. B.Sc. Sem-III (2014-Course): SUMMER-2018

SUBJECT : MATHEMATICS : CALCULUS OF SERVERAL VARIABLES

Day : **Tuesday**
Date : **24/04/2018**

S-2018-0713

Time : **12.00 NOON TO 02.00 PM**
Max. Marks : 40

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(10)**

a) Show that if $u = f(x,y)$ is a differentiable function of x and y and $x = \phi(t), y = \psi(t)$ are differentiable functions of t then the composite function $u = f\left[\phi(t), \psi(t)\right]$ is a differentiable function of t and its total derivative is given by

$$\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}.$$

b) If $f(x,y) = \dfrac{x^3 y}{x^2 + y^2}$, when $x^2 + y^2 \neq 0$ and $f(0,0) = 0$, then show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

c) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{x - y}\right)$ then prove that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \sin 2u$ and

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\sin u \cos 3u.$$

Q.2 Attempt any **TWO** of the following: **(10)**

a) State and prove Taylor's theorem for a function of two variables.

b) Bu using Taylors theorem, expand $x^2 + 2xy + yz + z^2$ about $(1,1,0)$.

c) Find the extreme value of the function

$$f(x,y) = xy + \frac{50}{x} + \frac{20}{y}$$

Q.3 Attempt any **TWO** of the following: **(10)**

a) Change the order of integration and hence evaluate

$$\int_0^1 \left[\int_x^1 e^{x/y} dy\right] dx$$

b) Find the area above the x-axis included between the curves $y^2 = x(2a - x)$ and $y^2 = ax$.

c) Evaluate $\iint_R \dfrac{dxdy}{xy}$, where R is the region in the xy-plane bounded by four circles $x^2 + y^2 = ax$, $x^2 + y^2 = bx$, $x^2 + y^2 = cy$ and $x^2 + y^2 = dy$ by using the transformation $\dfrac{x^2 + y^2}{x} = u$ and $\dfrac{x^2 + y^2}{y} = v$.

P.T.O.

Q.4 Attempt any **FIVE** of the following: **(10)**

a) Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4+y^2}$, along the path $y = mx$.

b) If $u = \dfrac{x^2+y^2}{x+y}$, then obtain $\dfrac{\partial u}{\partial x} - \dfrac{\partial u}{\partial y}$.

c) If $e^u = x^3 + y^3 - x^2 y - xy^2$ then find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$.

d) If $u = \dfrac{x+y}{1-xy}, v = \tan^{-1} x + \tan^{-1} y$ then find $\dfrac{\partial(u,v)}{\partial(x,y)}$.

e) State converse of Euler's theorem on homogeneous function.

f) Evaluate $\iint\limits_R (x \sin y - ye^x)\,dxdy$, where R is the rectangle $-1 \le x \le 1, 0 \le y \le \dfrac{\pi}{2}$.

g) Change the order of integration in $\displaystyle\int_0^1 \left[\int_y^{\sqrt{y}} f\,dx\right] dy$.

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