

F.Y.B.SC. SEM – II (CBCS - 2016 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS : INTEGRAL CALCULUS & DIFFERENTIAL EQUATIONS

Day : Saturday
Date : 28/04/2018

S-2018-0645

Time : 03.00 PM TO 06.00 PM
Max. Marks : 60

N.B.:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable **CALCULATOR** is allowed.

Q.1 A) Select the correct alternatives of the following:

[06]

i) $\int_0^{\pi/2} \cos^5 \theta d\theta = \underline{\hspace{2cm}}$.

- a) $\frac{8}{45}$ b) $\frac{16}{35}$ c) $\frac{8\pi}{30}$ d) $\frac{8}{15}$

ii) $\int_0^{\pi/2} \sin^2 \theta \cos^4 \theta d\theta = \underline{\hspace{2cm}}$.

- a) $\frac{5\pi}{128}$ b) $\frac{\pi}{64}$ c) $\frac{\pi}{32}$ d) $\frac{\pi}{16}$

iii) $\int \frac{dx}{\sqrt{x^2-1}} = \underline{\hspace{2cm}}$.

- a) $\sin^{-1} x + c$
b) $\log [x + \sqrt{x^2-1}] + c$
c) $\sec^{-1} x + c$
d) $\frac{x}{2} \sqrt{x^2-1} + \frac{1}{2} \log [x + \sqrt{x^2-1}] + c$

iv) Degree of the differential equation $\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2} = k \frac{d^2y}{dx^2}$ is $\underline{\hspace{2cm}}$.

- a) 2 b) 3 c) 1 d) $\frac{3}{2}$

v) Substitution for solving the differential equation $\frac{dy}{dx} = \frac{x^3 + y^3}{3xy^2}$, is $\underline{\hspace{2cm}}$.

- a) $v = x^3 + y^3$ b) $y = vx$ c) $v = xy^2$ d) $v = xy$

vi) If $Mdx + Ndy = 0$ is a homogeneous equation then its integrating factor is

- $\underline{\hspace{2cm}}$.
- a) $\frac{1}{Nx + My}$ if $Nx + My \neq 0$ b) $\frac{1}{Mx + Ny}$ if $Mx + Ny \neq 0$
c) $\frac{1}{Nx - My}$ if $Nx - My \neq 0$ d) $\frac{1}{Mx - Ny}$ if $Mx - Ny \neq 0$

P.T.O.

B) Solve the following: [06]

i) State the formula for obtaining length of the arc of the curve $r = f(\theta)$.

ii) State the formula for evaluating $\int_0^{\pi/2} \sin^m x \cos^n x dx$.

iii) Which substitution is used to evaluate $\int_0^2 x^3 \sqrt{2-x} dx$?

iv) Define exact differential equation.

v) Define integrating factor of the differential equation.

vi) Define orthogonal trajectory of one parameter family of curves.

Q.2 Attempt **ANY THREE** of the following: [12]

a) Evaluate: $\int \frac{x^2 - 1}{x^4 + 1} dx$.

b) Evaluate: $\int \frac{(x-1)(x-2)(x-3)}{(x+1)(x+2)(x+3)} dx$.

c) Evaluate: $\int \frac{(x+3) dx}{(x+1)(x^2+2)}$.

d) Find the orthogonal trajectories of the parabolas $y^2 = 4ax$.

Q.3 Attempt **ANY FOUR** of the following: [12]

a) Explain the method of solution of differential equation $\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$,
where c_1 and c_2 are not simultaneously zero when $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

b) Solve the differential equation $\frac{dy}{dx} = \frac{x+y-3}{x+y+3}$.

c) Solve the differential equation $(2x - y + 1) dx + (2y - x - 1) dy = 0$.

d) Evaluate: $\int_0^1 x^6 \sin^{-1} x dx$.

e) Evaluate: $\int \sec^4 x dx$.

Q.4 Attempt **ANY TWO** of the following: [12]

a) Prove that solution of the differential equation of the form $\frac{dy}{dx} + Py = Q$

where P and Q are functions of x alone is $ye^{\int P dx} = \int e^{\int P dx} Q dx + C$.

b) Solve: $(x^2y - 2xy^2) dx - (x^3 - 3x^2y) dy = 0$.

c) Solve: $xy - \frac{dy}{dx} = y^3 e^{-x^2}$.

Q.5 Attempt **ANY TWO** of the following: [12]

a) Evaluate: $\int \frac{dx}{a + b \sin x}$, if i) $a^2 > b^2$ ii) $a^2 < b^2$.

b) Find the length of the arc of the curve $x = t^2 \cos t, y = t^2 \sin t$ from the origin to the point t.

c) Find the volume of the solid generated by revolving the area included between the curves $y^2 = x^3$ and $x^2 = y^3$ about x - axis.