

S.Y.B.SC. SEM – III (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT: MATHEMATICS: GROUP THEORY & DIFFERENTIAL EQUATIONS

Day : **Monday**
Date : **30/04/2018**

S-2018-0658

Time : **03.00 PM TO 06.00 PM**
Max. Marks :60

N. B.:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(12)**

- a) Let G be a group. If $a, b \in G$ then prove that $(b^{-1}ab)^m = b^{-1}a^mb \forall m \in \mathbb{N}$.
- b) Let $Q_1 = \mathbb{Q} - \{1\}$ i.e, set of all rational numbers except 1 and $a*b = a + b - ab \forall a, b \in G$. Check whether $(Q_1, *)$ is group or not.
- c) Prove that the order of every element in a finite group is finite.

Q.2 Attempt any **TWO** of the following: **(12)**

- a) Show that intersection of two subgroups of a group is a subgroup. Is union of two subgroups of a group is a subgroup? Justify.
- b) Let 'a' be any element of a group G . We define $N(a) = \{x \in G / xa = ax\}$. Prove that $N(a)$ is a subgroup of G .
- c) Define subgroup, and find all the subgroups of a cyclic group of order 36.

Q.3 Attempt any **TWO** of the following: **(12)**

- a) Show that for the equation $f(D^2)y = \sin(ax + b)$ particular integral is given by $\frac{1}{f(D^2)} \sin(ax + b) = \frac{\sin(ax + b)}{f(-a^2)}$, where $f(-a^2) \neq 0$.
- b) Solve: $(D^2 - 1)y = x^2 \cos x$.
- c) Solve: $(D^4 + 2D^3 - 3D^2)y = x^2 + 3e^{2x}$.

P. T. O.

Q.4 Attempt any **THREE** of the following: **(12)**

- a) Solve the differential equation.
 $xp^2 - 2yp + x + 2y = 0$ using the substitution $y - x = v$ and $x^2 = u$.
- b) Solve: $e^{3x}(p-1) + p^3 e^{2y} = 0$, where $p = \frac{dy}{dx}$.
- c) Solve: $p^3 - 4xyp + 8y^2 = 0$, where $p = \frac{dy}{dx}$.
- d) Solve: $y = 2px + y^2 p^3$.

Q.5 Attempt any **FOUR** of the following: **(12)**

- a) In group G if every element is its own inverse then prove that G is abelian.
- b) Define order of an element in a group.
Let $S = \{ 1, -1, i, -i \}$ and (S, \cdot) be a group where \cdot is multiplication of complex numbers. Find order of every element of S.
- c) Prove that in any group there is unique identity element.
- d) Solve: $(D^3 + 7D^2 + 16D + 10)y = 0$.
- e) Find particular integral of $(D^3 + 4D)y = \cos 3x$.
- f) Solve: $\sin(y - xp) = p$, where $\frac{dy}{dx} = p$.

* * * *