S.Y.B.SC. SEM – IV (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT : MATHEMATICS: VECTOR CALCULUS

Day : **Tuesday** Time : **11.00 A.M. TO 02.00 PM**
Date : **24/04/2018** **S-2018-0670** Max. Marks : 60

N.B.:
1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[12]**

a) A differentiable vector function $\bar{u}(t)$ on [a, b] is of constant magnitude iff $\bar{u}.\frac{d\bar{u}}{dt} = 0, \forall t \in [a,b]$.

b) If $\bar{u} = 5t^2\bar{i} + t\bar{j} - t^3\bar{k}$ *and* $\bar{v} = \sin t\bar{i} - \cos t\bar{j}$, find :

i) $\frac{d}{dt}\left(\bar{u}.\bar{v}\right)$ **ii)** $\frac{d}{dt}\left(\bar{u} \times \bar{v}\right)$ **iii)** $\frac{d}{dt}\left(\bar{u}.\bar{u}\right)$

c) If $\bar{r} = 4a\sin\theta\bar{i} + 4a\cos\theta\bar{j} + 3b\cos 2\theta\bar{k}$, find:

i) $\left|\frac{d\bar{r}}{d\theta} \times \frac{d^2\bar{r}}{d\theta^2}\right|$ **ii)** $\left[\frac{d\bar{r}}{d\theta}, \frac{d^2\bar{r}}{d\theta^2}, \frac{d^3\bar{r}}{d\theta^3}\right]$

Q.2 Attempt **ANY TWO** of the following: **[12]**

a) If $\bar{f} = x^2yz\bar{i} - 2xz^3\bar{j} + xz^2\bar{k}$ and $\bar{g} = 2z\bar{i} + y\bar{j} - x^2\bar{k}$ then find $\frac{\partial^2}{\partial x \partial y}\left(\bar{f} \times \bar{g}\right)$ *at* $(1, 0, 2)$.

b) If $\bar{r} = x\cos y\bar{i} + y\sin y\bar{j} + ae^{my}\bar{k}$, find $\dfrac{\frac{\partial \bar{r}}{\partial x} \times \frac{\partial \bar{r}}{\partial y}}{\left|\frac{\partial \bar{r}}{\partial x} \times \frac{\partial \bar{r}}{\partial y}\right|}$.

c) Find the scalar function ϕ(x, y, z) if grad$\phi = y(2zx - 1)\bar{i} + x(xz - 1)\bar{j} + (x^2y + 4)\bar{k}$ and $\phi(2, 1, -1) = 0$

Q.3 Attempt **ANY TWO** of the following: **[12]**

a) If $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and $r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$, find:

i) ∇r **ii)** $div\,\bar{r}$ **iii)** $grad\left(r^n\right)$ **iv)** $curl\,\bar{r}$

b) Find the directional derivative of the function $\phi = x^2y^3 - 2xz^2 + 3$ at the point $P(2, 1, -2)$ in the direction towards $Q(4, 0, 3)$.

P.T.O.

c) Find the angle between the surfaces $x^2y + z = 3$ and $x\log z - y^2 + 4 = 0$, at the point (−1, 2, 1).

Q.4 Attempt **ANY THREE** of the following: **[12]**

a) Using Green's theorem, evaluate $\oint_C \left(e^{-x}\sin y\,dx + e^{-x}\cos y\,dy\right)$ where C is the rectangle formed by $x = 0,\ x = \pi,\ y = 0,\ y = \frac{\pi}{2}$.

b) If $\overline{f} = \left(3x^2 + 6y\right)\overline{i} - 14yz\overline{j} + 20xz^2\overline{k}$, evaluate $\int_C \overline{f}.\,d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the path C where $x = t,\ y = t^2,\ z = t^3$.

c) Using Stoke's theorem evaluate $\iint_S \left(\nabla\times\overline{f}\right).\,\overline{n}\,ds$ for $\overline{f} = (2x - y + z)\overline{i} + \left(x + y - z^2\right)\overline{j} + (3x - 2y + 4z)\overline{k}$ over the surface of cylinder $x^2 + y^2 = 4$ bounded by the plane z = 9 and open at end z = 0.

d) Prove that $curl\ (grad\phi) = \overline{o}$ that is $\nabla\times(\nabla\phi) = \overline{o}$.

Q.5 Attempt **ANY FOUR** of the following: **[12]**

a) Find the unit vector normal to the surface $x^2 + y^2 - z = 1$ at $(1, 1, 1)$.

b) If $\overline{r} = \left(t^2 + 1\right)\overline{i} + (4t - 3)\overline{j} + \left(2t^2 - 6t\right)\overline{k}$, find :

i) $\frac{d\overline{r}}{dt}$ ii) $\left|\frac{d\overline{r}}{dt}\right|$ iii) $\frac{d^2\overline{r}}{dt^2}$ at $t = 2$.

c) Show that $\overline{u} = 3y^4z^2\overline{i} + 4x^3z^2\overline{j} - 3x^2y^2\overline{k}$ is solenoidal.

d) Eliminate $\overline{a}$ and $\overline{b}$ from $\overline{r} = \overline{a}\cos 2t + \overline{b}\sin 2t$ and obtain the differential equation.

e) Evaluate $\int \overline{f}.d\overline{r}$ where $\overline{f} = y^2\overline{i} + 2xy\overline{j}$ from O(0, 0) to P(1, 1) along the straight line path OP.

f) Prove that $\nabla r^n = nr^{n-2}\overline{r}$, where $\overline{r} = x\overline{i} + y\overline{j} + z\overline{k}$.

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