F.Y.B.SC. SEM – I (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT : MATHEMATICS : CALCULUS

Day : **Wednesday** | Time : **11.00 A.M. TO 02.00 PM**
Date : **02/05/2018** | **S-2018-0631** | Max. Marks : 60

N. B. :
1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q. 1 A) Choose the correct alternatives of the following: **(06)**

i) If $y = \log(ax+b)$ *then* $y_n = \ldots\ldots\ldots$

a) $\dfrac{(-1)^{n-1}(n-1)!\,a^n}{(ax+b)^n}$ b) $\dfrac{(-1)^n\, n!\,a^n}{(ax+b)^n}$

c) $\dfrac{(-1)^n\, n!\,a^n}{(ax+b)^{n+1}}$ d) none of these

ii) A sequence $\{a_n\}$ where $a_n = 2(-1)^n - \dfrac{3}{n}$ *is*

a) convergent **b)** divergent
c) oscillatory **d)** none of these

iii) A series $\displaystyle\sum_{n=1}^{\infty} \frac{7-3n}{5n^2+3n+8}$ is

a) convergent **b)** divergent
c) oscillatory **d)** none of these

iv) If $f(x) = x^2$ satisfies conditions of Lagrange's mean value theorem over [1, 3] then by this theorem value of c is

a) 1.5 **b)** 1.2
c) 2 **d)** 2.5

v) $\displaystyle\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$ *is* ....

a) -1 **b)** 1
c) 1/2 **d)** 0

vi) $1 + x + \dfrac{x^2}{2!} + \dfrac{x^3}{3!} + \cdots + \dfrac{x^n}{n!} + \cdots\ \forall x \in R$, is expansion of

a) $\dfrac{1}{x}$ **b)** $\log x$
c) e^x **d)** None of these

B) Answer the following: **(06)**

i) Define removable and irremovable discontinuity.
ii) If $y = \sin 3x$ then find y_n
iii) State Heine's property.
iv) State geometrical meaning of Rolle's mean value theorem.
v) Define supremum of a function.
vi) Define uniform continuity of a function.

P. T. O.

Q. 2 Attempt **ANY THREE** of the following: **(12)**

a) State and prove Lagrange's mean value theorem.

b) Verify Cauchy's mean value theorem for the functions f (x) = x^2 and g (x) = x^4 and find the value of c over [2, 4].

c) Let $A(0, 1)$ and $B\left(\frac{\pi}{2}, 1\right)$ be two points on the locus given by $y = 2\sin x + \cos 2x$. Show that the tangent at P on the locus between A and B such that the tangent at P is parallel to the x-axis. Find the x – co-ordinate of the point P.

d) Verify Lagrange's mean value theorem for the function $f(x) = 2x^2 - 7x + 10$ over [2, 5].

Q. 3 Attempt **ANY FOUR** of the following: **(12)**

a) Show that if f is continuous on [a, b] and f (a) and f (b) have opposite signs then f (x) = 0, for some x ∈ [a, b].

b) Discuss the continuity of the following function in the interval [0, 8], where

$$f(x) = \frac{x^2+1}{x-2}, \text{ for } 0 \le x \le 3$$
$$= 3x - 1, \text{ for } 3 < x \le 5$$
$$= \frac{x^2+x+1}{x-1}, \text{ for } 5 < x \le 8.$$

c) Discuss the continuity of function f (x) if

$f(x) = \frac{e^{1/x}-1}{e^{1/x}+1}$, when $x \neq 0$ and $f(0) = 0$.

d) Show that the function $f(x)$ defined by $f(x) = |x|$ is continuous but not differentiable at $x = 0$.

e) Evaluate : $\lim_{x \to 0} x^x$.

Q. 4 Attempt **ANY TWO** of the following: **(12)**

a) Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$.

b) Show that a sequence whose n^{th} term is given below is monotonic and bounded, where

$$a_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \frac{1}{(n+3)^2} + \text{---} + \frac{1}{(n+n)^2}.$$

c) Discuss the convergence of the following series:

$i)\ \sum_{n=1}^{\infty} \frac{n!}{2^n}$ $\qquad$ $ii)\ \sum_{n=1}^{\infty} \frac{2n^2+3}{3n^4+1}$.

Q. 5 Attempt **ANY TWO** of the following: **(12)**

a) State and prove Leibnitz's theorem for n^{th} derivative of the product of two functions of x.

b) If $y = a\cos(\log x) + b\sin(\log x)$, then show that

$x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$.

c) Using Taylor's theorem expand $\tan x$, upto the term in x^5.

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