F.Y.B.SC. SEM – II (CBCS - 2016 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS : ANALYTICAL GEOMETRY

Day : **Wednesday** Time : **03.00 PM TO 06.00 PM**
Date : **25/04/2018** **S-2018-0643** Max. Marks : 60

N.B.

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is allowed.

Q.1 A) Choose the correct alternatives of the following: **(06)**

i) When the origin is shifted to the point (2, 1), the directions of axes remaining same, then the equation $xy - x - 2y + 2 = 0$ becomes

a) $XY - Y = 0$ **b)** $XY = 0$
c) $XY - Y = 2$ **d)** $X^2 - 2Y = 0$

ii) Equation of the line passing through the point $(-1, 2, 7)$ and having direction ratios 1,2,3 is

a) $\frac{x-1}{1} = \frac{y+2}{2} = \frac{z+7}{3}$ **b)** $\frac{x+1}{2} = \frac{y-2}{3} = \frac{z-7}{1}$
c) $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z-7}{3}$ **d)** None of these

iii) Direction ratios of normal to the plane $3x + 4y + 2 = 0$ are

a) 3, 4, 0 **b)** 3, 4, 2
c) –3, –4, 0 **d)** –3, –4, 2

iv) Centre of the sphere $x^2 + y^2 + z^2 - 2x + 4y - 6z + 11 = 0$ is

a) (1, 2, 3) **b)** (–1,2,–3)
c) (–1, –2, –3) **d)** (1,–2,3)

v) If the line $\frac{x-2}{-1} = \frac{y+3}{2} = \frac{z+4}{-k}$ is parallel to the plane $2x + 3y - 4z + 7 = 0$, then k = …..

a) 1 **b)** –1
c) 3 **d)** –3

vi) Direction ratios of the line joining the points A(1, –2, 3) and B(2, 3, –4) are

a) –1, –5, –1 **b)** –1,–5,–7
c) –1, –5, 7 **d)** 1, 5, 1

B) Solve the following: **(06)**

i) Identify the conic given by the equation
$11x^2 + 4xy + 14y^2 - 4x - 28y - 16 = 0$.

ii) Define direction cosines of a line.

iii) Find the equation of the plane passing through the point (1, 2, 3) and parallel to the plane $2x - y + 7z + 10 = 0$.

iv) Find the perpendicular distance of a point $P(x_1, y_1, z_1)$ from the plane $ax + by + cz + d = 0$.

v) Find the equation of the sphere whose centre is (1, –2, 3) and radius is 6.

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vi) Find the equation of the tangent plane to the sphere $x^2+y^2+z^2-6x-4y+10z=0$ at the origin

Q.2 Attempt any **THREE** of the following: **(12)**

a) Find the equations of the normal to the sphere $x^2+y^2+z^2+2ux+2vy+2wz+d=0$ at a point $P(x_1, y_1, z_1)$ on it.

b) Find the equation of the sphere through the circle $x^2+y^2+z^2=9$, $2x+3y+4z-5=0$

c) Find the symmetrical form the equations of the line of intersection of the planes $x+y+z+1=0$ and $4x+y-2z+2=0$

d) Find the equation of the plane through the points (–2, 1, 0) and perpendicular to the planes $4x-y+2z+5=0$ and $2x+4y+z-7=0$.

Q.3 Attempt any **FOUR** of the following: **(12)**

a) Prove that the general equation of first degree in x, y, z given by $ax+by+cz+d=0$, where a, b, c, d are constants(not all zero), represents a plane.

b) Show that the line $\frac{x-4}{1}=\frac{y-5}{2}=\frac{z+5}{-2}$ intersects the planes $3x-4y+5z-12=0$ and $5x+2y+z-4=0$ in the same point.

c) The centre of the sphere of radius 15 units is (3, b, –6). Find b if the point (–2, 2, 4) lies on the sphere; and find one equation of sphere.

d) Find the equation of the cone with vertex at the origin and containing the curve $x^2+y^2=4$ and $z=5$.

e) The origin is changed to the point (h, 2). Find the value of h so that the transformed equation of the locus given by $x^2+4x+3y-5=0$ will not contain a first degree term in x.

Q.4 Attempt any **TWO** of the following: **(12)**

a) Let O_x, O_y be the original frame of axes. If these axes are turned through an angle θ, so that the new frame of axes is Ox', Oy' then prove that $x = x'\cos\theta - y'\sin\theta$ and $y = x'\sin\theta + y'\cos\theta$.

b) Transform the equation $7x^2-8xy+y^2+14x-8y-2=0$ when the origin is shifted to the point (–1, 0) and then the axes are turned through an angle $\tan^{-1}\left(-\frac{1}{2}\right)$.

c) Find the centre and the lengths of axes of conic $x^2-3xy+y^2+10x-10y+21=0$

Q.5 Attempt any **TWO** of the following: **(12)**

a) Find the equation of the right circular cone with vertex at $V(\alpha, \beta, \gamma)$, semi-vertical angle θ and whose axis has direction ratios a, b, c.

b) The radius of a normal section of a right circular cylinder is 2 units. If its axis lies along the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-2}{5}$, find its equation.

c) The plane $\frac{x}{a}+\frac{y}{b}=\frac{z}{c}=1$ meets the co-ordinate axes in points A, B, and C. Prove that the equation of the cone generated by the lines drawn from the origin to meet the circle ABC is $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$

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