F.Y.B.SC. SEM – I (2014 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS : ALGEBRA (M – 11)

Day : **Wednesday**
Date : **25/04/2018**

S-2018-0687

Time : **12.00 NOON TO 02.00 PM**
Max. Marks : 40

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following: **[10]**

a) Prove that a necessary and sufficient condition for a square matrix A to have the inverse is that A is non-singular.

b) Find non-singular matrices P and Q such that PAQ is the normal form of A, hence determine rank of A, where $A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & -1 \end{bmatrix}$.

c) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$.

Q.2 Attempt **ANY TWO** of the following: **[10]**

a) Show that any two non-zero integers a and b have an unique (positive) g.c.d and this g.c.d can be expressed in the form $ma + nb$, where m, n $\in$ Z.

b) Find g.c.d 'd' of 3997 and 2947 and express it in the form d = $3997m + 3947n$ for some $m, n \in$ Z.

c) Test the consistency of the following system of equations and solve them if consistent $2x - y - z = 4$, $x - 2y + 4z = -1$, $3x - 4y + 6z = 1$.

Q.3 Attempt **ANY TWO** of the following: **[10]**

a) If z_1 and z_2 are any two complex numbers then prove that $|z_1 + z_2| \leq |z_1| + |z_2|$.

b) If $\cos\alpha + \cos\beta + \cos\gamma = 0$, $\sin\alpha + \sin\beta + \sin\gamma = 0$, then prove that:

i) $\cos 3\alpha + \cos 3\beta + \cos 3\gamma = 3\cos(\alpha + \beta + \gamma)$.

ii) $\sin 3\alpha + \sin 3\beta + \sin 3\gamma = 3\sin(\alpha + \beta + \gamma)$.

c) Solve the equation $x^7 + 1 = 0$, using De Moivre's theorem.

Q.4 Attempt **ANY FIVE** of the following: **[10]**

a) If $|z| = 1$ and arg z = θ, then prove that $\frac{1+z}{1-z} = i\cot\left(\frac{\theta}{2}\right)$.

b) Find the cube roots of unity.

c) Prove that if (a, b) = 1 and b | ac then b | c.

d) Find all the partitions of a set A = {3, 4, 5}.

e) If A is a non-singular matrix then prove that $(A')^{-1} = (A^{-1})'$.

f) Find eigen values of the matrix $A = \begin{bmatrix} -2 & 6 \\ 8 & -4 \end{bmatrix}$.

g) Define rank of a matrix.

* * * *