F.Y.B.SC. SEM – I (CBCS - 2016 COURSE) : SUMMER - 2018

SUBJECT : MATHEMATICS : ALGEBRA

Day : **Saturday**
Date : **28/04/2018**

S-2018-0629

Time : **11.00 A.M. TO 02.00 PM**
Max. Marks : 60

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 A) Select the correct alternatives of the following: **[06]**

i) If $A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$, then A^{-1} = ______.

a) $\frac{1}{5}\begin{bmatrix} -2 & 3 \\ 1 & -4 \end{bmatrix}$ **b)** $\frac{1}{5}\begin{bmatrix} 4 & -1 \\ -3 & 2 \end{bmatrix}$

c) $\frac{1}{5}\begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$ **d)** $\frac{1}{5}\begin{bmatrix} -4 & 1 \\ 3 & -2 \end{bmatrix}$

ii) If A is a non-zero matrix of order $m \times n$ then

a) $1 \le \text{rank } A \le \min\{m, n\}$ **b)** $1 \le \text{rank } A \le r$
c) rank A = 1 **d)** $1 \le \text{rank } A \le n$

iii) If $z = 3 + 4i$ then $z\bar{z}$ = ________.

a) 7 **b)** −7 **c)** 25 **d)** −25

iv) The value of $\frac{(\cos\theta + i\sin\theta)^3}{(\cos\theta - i\sin\theta)^5}$ = ________.

a) $\cos\theta + i\sin\theta$ **b)** $\cos 8\theta + i\sin 8\theta$
c) $\cos 2\theta + i\sin 2\theta$ **d)** $\cos 5\theta + i\sin 5\theta$

v) If g.c.d of a and b is denoted by (a, b) then (a, b) = ________.

a) (−a, b) **b)** (a, −b) **c)** (−a, −b) **d)** all of these

vi) If $b|a$ and $|a| < |b|$, then a = ________.

a) 1 **b)** b **c)** zero **d)** none of these

B) Answer the following questions: **[06]**

i) Define rank of a matrix.

ii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$.

iii) If A is a non-singular matrix then show that $|A^{-1}| = \frac{1}{|A|}$.

iv) State complex cube root of unity.

v) Show that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.

vi) Define g.c.d. of two integers a and b.

P.T.O.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Let a, b ∈ z, prove that if a|b, b ≠ 0 the |a| ≤ |b|. Further, if a is proper divisor of b then |a| < |b|.

b) Find the eigen values of the matrix $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -2 & 1 \\ 0 & -1 & 1 \end{bmatrix}$

c) Solve completely the system of equations
$2x + 3y + 4z + u = 0,\ 4x - 2y + z - 6u = 0,\ 6x + 5y + 3z - u = 0.$

d) Solve the equation $16x^4 + 8x^3 + 4x^2 + 2x + 1 = 0.$

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) State De Moivre's theorem and prove it for positive integers.

b) Find the continued product of four values of $\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right)^{3/4}$.

c) If |z| = 1 and arg z = θ, then prove that $\frac{1+z}{1-z} = i\cot\left(\frac{\theta}{2}\right)$.

d) Show that $\frac{(\cos 3\theta + i\sin 3\theta)^5 (\cos\theta - i\sin\theta)^3}{(\cos 5\theta + i\sin 5\theta)^7 (\cos 2\theta - i\sin 2\theta)^5} = \cos 13\theta - i\sin 13\theta$.

e) Find the fourth roots of unity.

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Prove that a necessary and sufficient condition for a square matrix A to have the inverse is that A is a non-singular matrix.

b) If $A = \begin{bmatrix} x-3 & 1 & 3 \\ 0 & x & 9 \\ -3 & 3 & x \end{bmatrix}$ prove that $\forall x \in R - \{3, \pm 3\sqrt{2}\},\ \rho(A) = 3$.
Find ρ(A) when $x = 3$.

c) Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Prove that any two non-zero integers a and b have an unique (positive) g.c.d. 'd' and can be expressed in the form, $d = (a, b) = ma + nb$, for some $m, n \in Z$.

b) Show that the integers 1357 and 1166 are relatively prime. Find integers m and n such that $1 = 1357m + 1166n$.

c) Investigate for what values of λ and μ the simultaneous equations
$x + y + z = 6,\ x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$ have (i) no solution (ii) an unique solution (iii) an infinite number of solutions.

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