S.Y.B.SC. SEM – III (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT: MATHEMATICS : CALCULUS OF SEVERAL VARIABLES

Day: **Friday** Time: **03.00 PM TO 06.00 PM**
Date: **27/04/2018** **S-2018-0656** Max. Marks: 60

N.B:

1) All questions are **COMPULSORY.**
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt any **TWO** of the following: **(12)**

a) Show that if $u = f(x, y)$ is a differentiable function of x and y and $x = \phi(t),\ y = \psi(t)$ are differentiatiable functions of t, then the composite function $u = f[\phi(t), \psi(\mathrm{t})]$ is a differentiable function of t and its total derivative is given by $\frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt}$.

b) If $f(x, y) = \frac{x^3 y}{x^2 + y^2}$ when $x^2 + y^2 \neq 0$ and $f(0,0) = 0$, then show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.

c) If $u = f(x, y)$ and $x = r\cos\theta, y = r\sin\theta$, then show that
$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$.

Q.2 Attempt any **TWO** of the following: **(12)**

a) State and prove Euler's theorem for a function of two variables x and y .

b) If $u = xy + yz + zx,\ v = x^2 + y^2 + z^2$ and $w = x + y + z,$ then show that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 0$.

c) Investigate the maximum and minimum values of
$f(x, \mathrm{y}) = (x + \mathrm{y} - 1)(x^2 + \mathrm{y}^2)$

Q.3 Attempt any**TWO** of the following: **(12)**

a) State and prove Taylor's theorem for a function of two variables x and y .

b) Using Tyalor's theorem expand $x^3 + y^3 + xy^2$ in powers of $(x-1)$ and $(y-2)$.

c) If a, b, c are positive numbers, find the extreme value of $f(x, y, z) = x^a y^b z^c$, subject to the side condition $x + y + z = 1$.

Q.4 Attempt any**THREE** of the following: **(12)**

a) Find the volume of tetrahedron bounded by the co-ordinate planes and the plane $x+y+z=1$.

b) Find by double integration the area of the reigion R in *xy*-plane bounded by $y^2=2x$ and $y=x$.

c) Change the order of integration and hence evaluate $\int_0^1\left[\int_x^1 e^{x/y}dy\right]dx$.

d) Four parabolas whose equations are $y^2=4ax,\ y^2=4bx,\ x^2=4cy,\ x^2=4dy$ intersect and form a quadrilateral space. Find the area of the space thus enclosed.

Q.5 Attempt any **FOUR** of the following: **(12)**

a) Prove that if f has an extremum at a point (a,b) and if the partial derivatives $f_x(x,y)$ and $f_y(x,y)$ exist in a neighbourhood of (a,b) then $f_x(a,b)=0\ =f_y(a,b)$.

b) Show that the following function is continuous at the origin, where $f(x,y)=\dfrac{x^3-y^3}{x^2+y^2}$ when $(x,y)\neq(0,0)$ and $f(0,0)=0$.

c) If $u=\tan^{-1}\left(\dfrac{y}{x}\right)$ then show that u is harmonic function.

d) If $u=x\phi\left(\dfrac{y}{x}\right)+\psi\left(\dfrac{y}{x}\right)$ then find the value of $x^2\dfrac{\partial^2u}{\partial x^2}+2xy\dfrac{\partial^2u}{\partial x\partial y}+y^2\dfrac{\partial^2u}{\partial y^2}$, given that ϕ and ψ are at least twice differentiable.

e) Evaluate $\iint_R xy(x+y)\,dxdy$, where R is the rectangle $0\le x\le1,\ 0\le y\le1$.

f) Change the order of integration $\int_0^2\left[\int_{2x}^{6-x} f\,dy\right]dx$.