S.Y. B. SC. (COMPUTER SCIENCE) SEM –III (CBCS - 2016 COURSE) : SUMMER - 2018

SUBJECT: LINEAR ALGEBRA

Day : **Thursday**
Date : **19/04/2018**

Time **03.00 PM TO 06.00 PM**
Max. Marks : 60

S-2018-0812

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.

Q.1 Attempt **ANY TWO** of the following. [12]

a) Prove that set S with two or more vectors is linearly dependent if and only if at least one of the vectors is expressible as a linear combination of other vectors in S.

b) Find a basis and dimension for the column space of matrix A where,

$$A=\begin{bmatrix} 2 & 3 & 5 & 7 & 4 \\ -1 & 2 & 1 & 0 & -2 \\ 4 & 1 & 5 & 9 & 8 \end{bmatrix}$$

c) Find the coordinate vector of $\bar{u} = (-1, 4, 1)$ relative to the basis, B = {(1,1,0), (1,0,1), (0,1,1)}

Q.2 Attempt **ANY TWO** of the following. [12]

a) Prove that if λ is an eigen value of a square matrix A, then λ^m is an eigen value of A^m for every positive integer m.

b) Find the eigen values of the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ also find eigen space corresponding to each eigen value of A. Further find basis and dimension of eigen space.

c) Determine whether A is diagonalizable where

$$A=\begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$ If so find $P^{-1}AP$.

Q.3 Attempt **ANY TWO** of the following. [12]

a) Let T:V $\rightarrow$ W be a linear transformation then prove that,
i) The kernel of T is a subspace of V.
ii) The range of T is a subspace of W.

b) If $T:\mathbb{R}^3 \rightarrow \mathbb{R}^3$ is a linear transformation defined by
T (x, y, z) = (x + 2y + 3z, 2x + 3y + 4z, 3x + 4y + 5z)
Find: i) rank(T) and ii) Nullity (T). Also verify dimension theorem.

c) Find the standard matrix for the linear transformation $T:\mathbb{R}^5 \rightarrow \mathbb{R}^2$ is defined by, $T(x_1, x_2, x_3, x_4, x_5) = (3x_1 - x_2 + 4x_3, 3x_2 + 4x_4 - 2x_5)$

P.T.O.

Q.4 Attempt **ANY THREE** of the following. [12]

a) For what value of 'a' the linear system has
i) no solution
ii) a unique solution
iii) Infinitely many solutions.

$$x + y - z = 2$$
$$x + 2y + z = 3$$
$$x + y + (a^2 - 5)z = a$$

b) Solve the system:

$$x_1 + 2x_2 + x_3 = 1$$
$$2x_1 - x_2 + x_3 = 2$$
$$4x_1 + 3x_2 + 3x_3 = 4$$
$$3x_1 + x_2 + 2x_3 = 3$$

c) Reduce the following matrix into reduced row echelon form

$$A = \begin{bmatrix} 2 & 1 & 0 & 5 \\ 3 & 6 & 1 & 1 \\ 5 & 7 & 1 & 8 \end{bmatrix}$$

d) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is any 2x2 matrix, then show that if A is invertible,

then ad $-$bc $\neq 0$ and in this case $A^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Q.5 Attempt **ANY FOUR** of the following: [12]

a) Define: i) Vector space
ii) Subspace of a vector space

b) Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by T (x, y, z) = (x, y, 0) Determine whether T is linear transformation?

c) If $T: \mathbb{R}^2 \to \mathbb{R}^2$ is linear transformation defined by T (x, y) = (x, x + y − 1).
Find: i) Kernel T.
ii) Range T.

d) State True of False:
"Every system of n linear equation in n unknowns has a unique solution". Justify your answer.

e) Find all eigen values of matrix A and hence, write the eigen values of A^t and A^{-1}, where $A = \begin{bmatrix} 3 & 2 & 2 \\ 1 & 4 & 1 \\ -2 & -4 & -1 \end{bmatrix}$

f) Find the dot product of $\bar{u} = \begin{bmatrix} 2 \\ -3 \\ 5 \\ 4 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 3 \\ 1 \\ -2 \\ 4 \end{bmatrix}$.

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