B.SC. (I. T.) SEM. - I (2011 COURSE) : SUMMER - 2018
SUBJECT : DISCRETE MATHEMATICS

Day : **Friday**
Date : **25/05/2018**

S-2018-0969

Time : **02.30 pm to 05.30 pm**
Max. Marks : 80.

N.B.:
1) Attempt any **FIVE** questions.
2) Figures to the **RIGHT** indicate full marks.

Q. 1 a) Prove that $(A \cup B)\backslash (A \cap B) = (A\backslash B) \cup (B\backslash A)$ **(04)**

b) Determine the validity of the following argument. **(04)**
If 7 is less than 4, then 7 is not a prime number
7 is not less than 4
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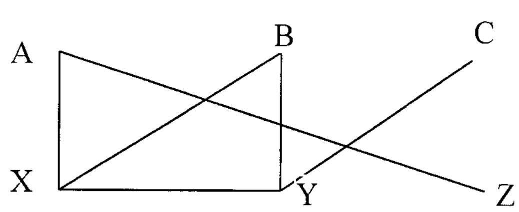
7 is a prime number

c) Given that A = {1, 2, 3, 4} and B= {x, y, z}. Let R be the following relation from A to B: **(08)**
R = { (1, y), (1, z), (3, y), (4, x), (4, z) }
i) Determine the matrix of the relation
ii) Draw the arrow diagram of R
iii) Find the inverse relation R^{-1}.
iv) Determine the domain and range of R

Q.2 a) Prove that $\sqrt{2}$ is not a rational number. **(04)**

b) A survey of sample of 25 new cars being sold at a local auto dealer showed that 15 had Air-conditioning (A), 12 had Radio (R), 11 had Power windows (P). If 5 had A and P, 9 had A and R, 4 had R and P and 3 had all options, find the number of cars that had **(06)**
i) Only P.
ii) R and P but not A
iii) None of the options

c) For the graph in the following figure, find: **(06)**
i) All simple paths from A to C
ii) All cycles
iii) All bridges
iv) Sub graph generated by V' = {B, C, X, Y}



Q. 3 a) Consider the following relations on A = {1, 2, 3}. **(10)**
R = { (1, 1), (1, 2), (1, 3), (3, 3)}
S = { (1, 1), (1, 2), (2, 1), (2, 2)}
T = { (1, 1), (1, 2), (2, 2), (2, 3)}
φ = empty relation; $A \times A = Universal\ relation.$

P.T.O.

Determine whether or not , each of the above relations is

i) Reflexive

ii) Symmetric

iii) Transitive

iv) Anti-symmetric

b) Consider the adjacency matrix given below: **(06)**

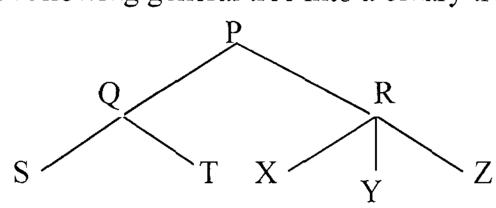
$$A = \begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

i) How many vertices does the graph A have?

ii) Are there any loops? Briefly explain.

iii) Draw the graph.

Q. 4 a) Convert the following general tree into a binary tree. **(06)**



b) Find the pre-order, in-order and post-order traversals for the binary tree in a) above. **(06)**

c) Let T be binary tree, with n vertices, find minimum and maximum possible height of the tree. **(04)**

Q. 5 a) Prove by the principle of mathematical induction that for all $n \in N$ **(06)**

$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{1}{6}\, n(n+1)(2n+1)$

b) Six boys and six girls are to be seated in a row. In how many ways can they be seated if **(06)**

i) All boys and all girls are to be seated together.

ii) No two girls are to be seated together.

c) Consider a function $f: R \rightarrow R$ given by $f(x) = 3x^2 + 5, \forall x \in R$. Show that f is bijective. **(04)**

Q.6 a) Prove that **(06)**

i) ${}^{n}C_{r} = {}^{n-1}C_{r} + {}^{n-1}C_{r-1}$

ii) ${}^{n}C_{r} = {}^{n}C_{n-r}$

b) Verify the proposition $p \vee \neg(p \wedge q)$ is a tautology. **(06)**

c) Define the following and draw one example for each: **(04)**

i) Path

ii) Trail

iii) Cycle

iv) Cut Point

Q. 7 a) Prove that tree with n vertices has (n – 1) edges. **(06)**

b) Among first 500 positive integers **(06)**

i) Determine the integer which is not divisible by 2 nor 3.

ii) Determine the integers which are exactly divisible by one of them.

c) State De Morgan's law and illustrate using Venn diagram. **(04)**

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