F.Y. B. SC. (COMPUTER SCIENCE) SEM –II (CBCS - 2016 COURSE) : SUMMER - 2018
SUBJECT : ALGEBRA – II

Day : **Wednesday** Time : **03.00 PM TO 06.00 PM**
Date : **18/04/2018** Max. Marks : 60

S-2018-0804

N.B.:

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of statistical tables and **CALCULATOR** is allowed.

Q.1 A) Select the correct option and rewrite complete sentence: **[06]**

i) ϕ (24) = __________.
a) 2 **b)** 4 **c)** 8 **d)** 10

ii) The order of $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 4 & 5 & 3 & 7 & 6 \end{pmatrix} \in s_7$.
a) 3 **b)** 6 **c)** 9 **d)** 12

iii) The number of subgroup of group G of order 51 is _________.
a) 2 **b)** 1 **c)** 3 **d)** 6

iv) The order of symmetric group on 4 letters is __________.
a) 24 **b)** 4 **c)** 12 **d)** 36

v) The order of element $\bar{4}$ in $(Z_6, +_6)$ _____.
a) 2 **b)** 4 **c)** 3 **d)** None of these

vi) Generators of group G = $\{1, \omega, \omega^2\}$ are _________.
a) $\{1, \omega\}$ **b)** $\{\omega, \omega^2\}$ **c)** $\{1, \omega^2\}$ **d)** None of these

B) Attempt the following: **[06]**

i) State Lagrange's theorem.

ii) Define Integral Domain.

iii) Define cyclic group with example.

iv) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$, express σ as a product of disjoint cycles.

v) Define Ring.

vi) Find $\bar{4} \times_5 \bar{2}$ in Z_5.

Q.2 Attempt **ANY THREE** of the following: **[12]**

a) Prove that $O(a^{-1}) = O(a)$, if a be an element in a group G.

b) Express the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 2 & 5 & 4 & 3 & 6 & 1 & 7 & 9 & 8 \end{pmatrix}$. Determine whether σ is even or odd. Find σ^{-1}.

c) Show that (Z, +) is isomorphic to (mZ, +).

d) Write down factor group $\frac{z_6}{\langle \bar{2} \rangle}$.

P.T.O.

Q.3 Attempt **ANY FOUR** of the following: **[12]**

a) Prepare multiplication table for Z_8^*, set of all prime residue classes modulo 8.

b) Find the order of every element in $(Z_6, +_6)$.

c) Let f: (Z, +) $\rightarrow$ (G, .) be defined by $f(n) = i^n$, where G = {1, -1, i, -i}. Show that f is homomorphism and also find its Kernel.

d) Show that identify element in G is unique.

e) Find all solutions of $x^2 - 5x + 6 = 0$ in Z_{12}.

Q.4 Attempt **ANY TWO** of the following: **[12]**

a) Show that (G, *) is a group. Is it abelian? Justify if G be the set of all integers. Define * as follows for a, b $\in$ G, a * b = a + b – 2.

b) State and prove left cancellation law and right cancellation law.

c) Show that A_3 is normal in S_3.

Q.5 Attempt **ANY TWO** of the following: **[12]**

a) Prove that subgroup H of group G is normal if and only if $xHx^{-1} = H \ \forall x \in G$.

b) State and prove Reversal Law.

c) Given $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ and $h = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 2 & 4 & 1 \end{pmatrix}$

Find :

i) f^{-1} **ii)** g^{-1} **iii)** h^{-1} **iv)** fog **v)** gof^{-1} **vi)** $h^{-1}og^{-1}$.

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