

**B. Tech. SEM -II (Computer Science & Business Systems) (CBCS
2018 Course) : SUMMER - 2019
SUBJECT : MATHEMATICS – II**

Day Wednesday
Date 22/05/2019

Time : 10.00 AM To 01.00 PM
Max. Marks : 60

S-2019-2519

N.B.:

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Use of non-programmable **CALCULATOR** is allowed.
- 4) Assume suitable data if necessary.

Q.1 a) By using properties of determinants, prove that : **[05]**

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

b) Solve the following system of equations using Cramer's rule : **[05]**

$$\begin{aligned} x - 4y - z &= 11, \\ 2x - 5y + 2z &= 39, \\ -3x + 2y + z &= 1 \end{aligned}$$

OR

a) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, verify that $(AB)C = A(BC)$. **[05]**

b) For $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{bmatrix}$, find adjoint (A) and A^{-1} . **[05]**

Q.2 a) Reduce the following matrix A to its normal form and hence find its rank **[05]**
where:

$$A = \begin{bmatrix} 2 & -1 & 1 & 3 \\ 2 & 4 & -1 & 0 \\ 4 & 3 & 2 & 1 \end{bmatrix}$$

b) Apply Gauss - elimination method to solve the equation: **[05]**

$$\begin{aligned} x + 4y - z &= -5, \\ x + y - 6z &= -12, \\ 3x - y - z &= 4 \end{aligned}$$

OR

Solve the system of equations by LU decomposition method: **[10]**

$$\begin{aligned} 2x_1 + 2x_2 + 3x_3 &= 4, \\ 4x_1 - 2x_2 + x_3 &= 9, \\ x_1 + 5x_2 + 4x_3 &= 3 \end{aligned}$$

Q.3 a) Let V be the set of all ordered pairs of real numbers, and consider the following **[05]**
addition and scalar multiplication operations on

$$u = (u_1, u_2) \text{ and } v = (v_1, v_2):$$

$$u + v = (u_1 + v_1, u_2 + v_2), \quad ku = (ku_1, 0) \text{ where } k \text{ is any real number}$$

- i) Compute $u + v$ and ku for $u = (2, 4)$, $v = (-3, 5)$ and $K = 7$.
- ii) Show that V is not a vector space under the given operations.

P.T.O.

- b) Determine whether the vectors span \mathbb{R}^3 . [05]
 $V_1 = (2, -1, 3)$, $V_2 = (4, 1, 2)$, $V_3 = (8, -1, 8)$.

OR

- a) Find a basis for $\langle S \rangle$, where : [05]

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \\ 3 \end{bmatrix} \right\}$$

- b) Let L be the line through the origin in \mathbb{R}^2 that is parallel to the vector $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Find [05]
the standard matrix of the orthogonal projection onto L and also find the point on L which is closest to the point (7, 1).

- Q.4 Let \mathbb{R}^3 have the inner product $\langle u, v \rangle = u_1 v_1 + 2u_2 v_2 + 3u_3 v_3$. [10]
Use the Gram-Schmidt process to transform
 $u_1 = (1, 1, 1)$, $u_2 = (1, 1, 0)$, $u_3 = (1, 0, 0)$ into an orthonormal basis.

OR

Find the QR – decomposition of the matrix $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \\ 0 & 3 & 1 \end{bmatrix}$. [10]

- Q.5 Find Eigen values and Eigen vectors for the matrix. [10]

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

OR

- a) Consider the following two bases of \mathbb{R}^2 : [05]
 $S = \{u_1, u_2\} = \{(1, 2), (3, 5)\}$ and
 $S' = \{v_1, v_2\} = \{(1, -1), (1, -2)\}$
Find the change of basis matrix P from S to the “new” basis S'.

- b) If $A = \begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$, show that A is a Hermitian matrix. [05]

- Q.6 Find the singular value decomposition of $A = \begin{bmatrix} 5 & 5 \\ -1 & 7 \end{bmatrix}$. [10]

OR

What is PCA? Write the objectives of PCA and explain PCA with example. [10]

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