B. Tech. SEM -I (Computer Science & Business Systems) (CBCS 2018 Course) : SUMMER - 2019

SUBJECT : MATHEMATICS – I

Day : Thursday
Date : 09/05/2019

S-2019-2513

Time : 10.00 AM To 01.00 PM
Max. Marks : 60

N. B. :

1) All questions are **COMPULSORY**.
2) Figures to the right indicate **FULL** marks.
3) Use of non-programmable calculator is **ALLOWED**.

Q. 1 Evaluate the following integrals:

a) $\iint_R y\,dxdy$, where R is $y = x^2,\ x + y = 2,\ x = 0$ in first quadrant. **(05)**

b) $\int_0^1 \int_0^{\sqrt{(1-y^2)}} \frac{dxdy}{\sqrt{1 - x^2 - y^2}}$. **(05)**

OR

Change the order of integration in the double integral **(10)**

$$\int_0^a \int_{-a+\sqrt{a^2-y^2}}^{a+\sqrt{a^2-y^2}} f(x,y)\ \text{dxdy}.$$

Q. 2 **a)** Find the volume of the region enclosed by the cone **(05)**

$z = \sqrt{x^2 + y^2}$ and paraboloid $z = x^2 + y^2$.

b) Find the total area included between the two cardioids $r = a\,(1 + \cos\theta)$ and $r = a\,(1 - \cos\theta)$. **(05)**

OR

Find the area inside the cardioid $r = 2a\,(1 + \cos\theta)$ and outside the parabola $r = \frac{2a}{1 + \cos\theta}$. **(10)**

Q. 3 **a)** Among the first 1000 positive integers determine the integers which are not divisible by 5 nor by 7 nor by 9. **(05)**

b) Construct the truth table for the following statement: **(05)**

$(\sim q \to \sim p) \to (p \to q)$.

OR

a) Determine whether the relation **(05)**

$R = \{(a, b) \in R,\ a - b \le 1, \text{on the set of positive integers}\}$ is an equivalence relation.

b) Construct the truth table for the following statement: **(05)**

$p \leftrightarrow (\sim p \vee \sim q)$.

P. T. O.

Q. 4 **a)** Reduce the expression **(05)**

$$\overline{XY} + \bar{X} + XY.$$

b) Design a circuit to realize the following: **(05)**

$$F(A, B, C) = AB + A\bar{C} + \bar{B}\bar{A}C.$$

OR

What is the simplified Boolean equation for the function: **(10)**

$$F(A, B, C, D) = \Sigma(7, 9, 10, 11, 12, 13, 14, 15).$$

Q. 5 **a)** Consider the binary operation $*$ on I_+ (the set of positive integers) defined by $a*b = \frac{ab}{2}\ \forall a, b \in I_+$. Determine the identity for the binary operation $*$, if exists. **(05)**

b) Let $(G, *)$ be a group, where G is a set having elements $\{0, 1\}$ and $*$ is a binary operation. Also, let $H = \{1\}$ is a subgroup of G. Determine all the left coset of H in G. **(05)**

OR

Consider an algebraic system $(Q, *)$, where Q is the set of rational numbers and $*$ is a binary operation defined by $a*b = a + b - ab\ \forall\ a, b \in Q$. Determine whether $(Q, *)$ is a group. **(10)**

Q. 6 **a)** Determine the discrete numeric functions corresponding to the following generation function: **(05)**

$$A(Z) = \frac{2 + 3Z - 6Z^2}{1 - 2Z}.$$

b) Prove the following by mathematical induction: **(05)**

$$1.2 + 2.3 + 3.4 + - - - + n.(n+1) = \left[\frac{n(n+1)(n+2)}{3}\right].$$

OR

a) Solve: $a_r - a_{r-1} - 6a_{r-2} = -30$ given $a_0 = 20, a_1 = -5.$ **(05)**

b) Show that for any integer n, **(05)**

$$11^{n+2} + 12^{2n+1} \text{ is divisible by } 133.$$

* * * * *