B.Tech Sem – IV (2007 Course) (Mechanical Engg.)/ (Producation Engg.) : SUMMER - 2019

SUBJECT : ENGINEERING MATHEMATICS – III

Day : Thursday

Date : 23/05/2019

Time : 10.00 AM TO 01.00 PM

Max. Marks : 80

S-2019-3033

N.B.:

1) **Q.No.1** and **Q.No.5** are **COMPULSORY**. Out of the remaining questions attempt **ANY TWO** questions from each section.
2) Answer to both the sections should be written in **SAME** Answer book.
3) Figures to the right indicate **FULL** marks.
4) Use of non-programmable **CALCULATOR** is allowed.
5) Assume suitable data if necessary.

SECTION – I

Q.1 a) Solve : $\dfrac{dx}{2x} = \dfrac{dy}{-y} = \dfrac{dz}{4xy^2 - 2z}$ **[04]**

b) Find the Fourier cosine transform of the function $f(x) = 4e^{-3x}$. **[05]**

c) An electric current consists of an inductance 0.1 henry, a resistance R of 20 ohms and a condenser of capacitance C of 25 microfarads. If the differential equation of electric circuit is $L\dfrac{d^2q}{dt^2} + R\dfrac{dq}{dt} + \dfrac{q}{C} = 0$ then find the charge q and current i at any time t, given that, at t = 0, q = 0.05 coulombs, $i = \dfrac{dq}{dt} = 0$ when $t = 0$. **[05]**

Q.2 Solve **ANY THREE**: **[13]**

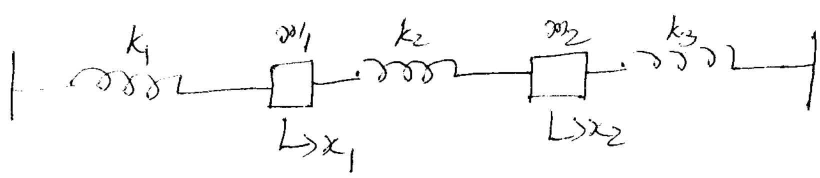
a) $(D^2 - 7D + 6)y = e^{2x}$

b) $(D^3 + D)y = \cos x$

c) $x^2\dfrac{d^2y}{dx^2} - 4x\dfrac{dy}{dx} + 6y = x^5$

d) $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = \sin(e^x)$ (by variation of parameter method)

Q.3 a) For the system shown in adjoining figure if $m_1 = 1$, $m_2 = 3$, $k_1 = 1$, $k_2 = 3$, $k_3 = 3$, assuming that there is no friction, find the natural frequencies of the system and corresponding normal modes of vibration using matrix method. **[07]**



b) Solve $\dfrac{\partial U}{\partial t} = \dfrac{\partial^2 U}{\partial x^2}$, if **[06]**

i) U is finite $\forall$t

ii) U = 0 when x = 0, $\forall$t

iii) U = 0 when $x = \pi$, $\forall$t

iv) $U = \pi x - x^2$, when t = 0 and $0 \le x \le \pi$

P.T.O.

Q.4 **a)** Find Laplace transform of $\int_0^t \frac{e^{-at}\sin bt}{t}\,dt$. **[04]**

b) Find inverse Laplace transform of $\frac{11s^2-2s+5}{(s-2)(2s-1)(s+1)}$. **[05]**

c) Find the Fourier transform of $f(x)=\begin{cases} x & , \quad 0\le x\le 1/2 \\ 1-x & , \quad 1/2\le x\le 1 \\ 0 & , \quad x>1 \end{cases}$ **[04]**

SECTION – II

Q.5 **a)** A dice is thrown 6 times. If "getting an even number" is a "success". What is the probability of exactly 5 successes? **[05]**

b) Find the directional derivative of $\phi = xy^2 + yz^3$ at (1, −1, 1) towards the point (3, 1, −1). **[04]**

c) Evaluate $\oint_C \left(\cos y\,\vec{i} + x(1-\sin y)\,\vec{j}\right).\,d\vec{r}$ for a closed curve which is given by $x^2+y^2=1,\ z=0$ **[05]**

Q.6 **a)** Find the coefficient of correlation between x and y from the following table. **[05]**

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

b) From a deck of 52 cards, two cards are drawn at random. Find the probability that both the cards are of different suits. **[04]**

c) A manufacture of cotter pins know that 3% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that not more than 6 pins will be defective in a box. Find the probability that a box will fail to meet the guaranteed quality. **[04]**

Q.7 **a)** A particle describes the curve r = 2a cos θ with constant angular speed ω. Find radial and transverse components of velocity and acceleration. **[05]**

b) Show that : $\nabla^4(\log r) = \frac{2}{r^4}$ **[04]**

c) Show that the vector filed $\vec{F} = \left(y^2\cos x + z^2\right)\vec{i} + 2y\sin x\,\vec{j} + 2xz\,\vec{k}$ is irrotational. **[04]**

Q.8 **a)** Evaluate $\iint_S \left(\nabla\times\vec{F}\right).\,d\vec{s}$, where $\vec{F} = \left(x^3-y^3\right)\vec{i} - xyz\,\vec{j} + y^3\vec{k}$ and s is the surface $x^2+4y^2+z^2-2x=4$ above the plane x = 0. **[06]**

b) Verify the divergence theorem for the function $\vec{F} = x\vec{i} + y\vec{j} + z^2\vec{k}$ over the cylindrical region bounded by $x^2+y^2=4,\ z=0,\ z=2$. **[07]**

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