

Day : Thursday
Date : 09/05/2019

S-2019-2561

Time : 02.30 PM TO 05.30 PM
Max. Marks : 60

N. B. :

- 1) All questions are **COMPULSORY**.
- 2) Figures to the right indicate **FULL** marks.
- 3) Draw neat and labeled diagram **WHEREVER** necessary.
- 4) Use on non-programmable calculator is **ALLOWED**.
- 5) Assume suitable data, if necessary.

Q. 1 Solve : $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z - a\sqrt{x^2 + y^2 + z^2}}$ (10)

OR

Solve : (10)

$$\frac{dx}{dt} + 5x - 2y = t$$
$$\frac{dy}{dt} + 2x + y = 0$$

given that $x = 0; y = 0$ at $t = 0$.

Q. 2 Evaluate : (10)

$$\int_c \frac{(4z^3 - 1) dz}{(z^2 - 1)(z + 1)^3(z - 4)} \quad \text{where } c : |z| = \frac{5}{2}$$

OR

Find the bilinear transformation which maps the points $(-i, 0, 2 + i)$ of the Z -plane onto the points $(0, -2i, 4)$ of the W -plane. (10)

Q. 3 a) Find the z -transform of $(k + 1)(k + 3)4^k; k \geq 0$. (05)

b) Find Fourier cosine transform of x^{n-1} (05)

OR

Find Fourier transform of $\frac{e^{-ax}}{x}$ and hence evaluate $\int_0^{\infty} \tan^{-1}\left(\frac{x}{a}\right) \sin x dx$ (10)

P. T. O.

Q. 4 a) Evaluate, using Laplace transform: (05)

$$\int_0^{\infty} e^{-at} \frac{\sin t}{t} dt.$$

b) Find : (05)

$$L^{-1} \left[\log \left(\frac{s+4}{s+7} \right) \right]$$

OR

Solve using inverse Laplace transform : (10)

$$\frac{d^2x}{dt^2} + 9x = \cos 2t, \text{ if } x(0) = 1, x\left(\frac{\pi}{2}\right) = -1.$$

Q. 5 a) Find the function $g(r)$ so that $g(r)\bar{r}$ is solenoidal. (05)

b) Find the value of $\nabla^2(r^n \log r)$. (05)

OR

Show that vector field : (10)

$\bar{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational. Find the scalar potential ψ such that $\bar{F} = \nabla\psi$

Q. 6 Evaluate : $\iint_S (x^3\bar{i} + y^3\bar{j} + z^3\bar{k}) \cdot ds$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. (10)

OR

State and prove Green's theorem. (10)

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