Day: Thursday   **S-2019-2525**   Time: 10.00 AM TO 01.00 PM
Date: 09/05/2019   Max Marks. 60

N.B. :
1)   All questions are **COMPURSORY**.
2)   Figures to the right indicate **FULL** marks.
3)   Use of non-programmable calculator is **ALLOWED**.
4)   Draw neat and labeled diagram **WHEREVER** necessary.

Q.1   **a)**   Find the rank of the following matrix   **(05)**

$$\begin{bmatrix} 3 & 2 & -1 & 5 \\ 5 & 1 & 4 & -2 \\ 1 & -4 & 11 & -19 \end{bmatrix}$$

      b)   Examine for linear dependence or independence for the following system of   **(05)** vectors.

$$x_1 = (3, 1, -4), \ x_2 = (2, 2, -3), \ x_3 = (0, -4, 1)$$

OR

Q.1   Find the eigen values and the corresponding eigen vectors of the following   **(10)** matrix.

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Q.2   **a)**   A square lies entirely in second quadrant if one of the side join the points -2   **(05)** and 2i find the complex numbers representing other vertices.

      b)   If Z_1, Z_2, Z_3 represent vertices of an equilateral triangle, prove that   **(05)**

$$Z_1^2 + Z_2^2 + Z_3^2 = Z_1 Z_2 + Z_2 Z_3 + Z_3 Z_1$$

OR

Q.2   **a)**   Find the value of $\tanh\left(\log(\sqrt{5})\right)$ **(05)**

      b)   If $\tan(A + iB) = x + iy$ then prove that $x^2 + y^2 + 2x \cot 2A = 1$ **(05)**

Q.3   **a)**   If $y = \cos(m \log x)$, show that $x^2 y_{n+2} + (2n+1) xy_{n+1} + (m^2 + n^2) y_n = 0$ **(05)**

      b)   If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that $(x^2 - 1) y_{n+2} + (2n+1) xy_{n+1} + (n^2 - m^2) y_n = 0$

OR

Q.3   Prove that the value of the nth differential coefficient of $\dfrac{x^3}{(x^2 - 1)}$ for $x = 0$ is   **(10)** zero if n is even and $-n!$ if n is odd and greater than 1

P.T.O.

Q.4 **a)** Evaluate $\lim\limits_{x\to 0} \dfrac{e^x\sin x - x - x^2}{x^2 + x\log(1-x)}$ **(05)**

b) Test the convergence of the series: **(05)**
$$\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \dots$$

OR

Q.4 **a)** Test the convergence of the series: **(05)**
$$1 + \frac{1}{2.3} + \frac{1.3}{2.4.5} + \frac{1.3.5}{2.4.6.7} + \dots$$

b) Find a, b, c if $\lim\limits_{x\to 0} \dfrac{ae^x - be^{-x} - cx}{x - \sin x} = 4$ **(05)**

Q.5 **a)** If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$ then prove that **(05)**
$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$$

b) If $u = \tan^{-1}\left(\dfrac{x^3 + y^3}{2x + 3y}\right)$ then prove that **(05)**
$$x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = \sin 2u(2\cos 2u - 1)$$

OR

Q.5 Find $\dfrac{dy}{dx} + \dfrac{d^2y}{dx^2}$ for $x^4 + y^4 = 5a^2 xy$ **(10)**

Q.6 **a)** If $u^3 + v^3 + w^3 = x + y + z$, $u^2 + v^2 + w^2 = x^3 + y^3 + z^3$, $u + v + w = x^2 + y^2 + z^2$ **(05)** then find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$

b) If the kinetic energy is given by $T = \dfrac{mv^2}{2}$, find approximately the change in **(05)** T as m changes from 49 to 49.5 and v changes from 1600 to 1590.

OR

Q.6 Examine for minimum and maximum values for **(10)**
$$\sin x + \sin y + \sin(x + y)$$

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